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Finite-time boundedness, L_2-gain analysis and control of Markovian jump switched neural networks with additive time-varying delays

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ABSTRACT

This paper is concerned with the problem of finite-time boundedness, L_2 gain analysis and control of Markovian jump switched neural networks (MJSNNs) with additive time-varying delays. Sufficient conditions to guarantee finite-time bounded of MJSNNs with additive time-varying delays are presented. These conditions are delay-dependent and are given in terms of linear matrix inequalities (LMIs). Moreover, finite-time L_2-gain analysis of switched neural networks with additive time-varying delays are given to measure its disturbance tolerance capability in the fixed time interval. In addition finite-time control of MJSNNs is studied. Detailed proofs are accomplished by using Lyapunov-functionals and average dwell time approach (ADT). Finally, numerical example is given to demonstrate the effectiveness of the proposed approach.

1. Introduction

In recent years, neural networks (especially switched neural networks, recurrent neural networks, Hopfield neural networks and cellular neural networks) have been successfully applied in many areas such as pattern recognition, associative memory, image processing, fault diagnosis, and combinatorial optimization. Many papers have focused on studying the existence, uniqueness, and global robust asymptotic stability of the equilibrium point in the presence of time delays and parameter uncertainties for various classes of nonlinear neural networks (see [1–8]).

Switched neural networks (SNNs), whose individual sub-systems are a set of neural networks, have attracted significant attention and have been successfully applied to many fields such as high-speed signal processing, artificial intelligence and gene selection in DNA micro array analysis [9–11]. Recent researches in switched time-delay system typically focus on the analysis of dynamic behaviors, such as stability, controllability, reachability, and observability aiming to design controllers with guaranteed stability and performance [12–17].

Recently, the authors in (see [18–23]) reported that the signals transmitted, in the network control system, from one point to another pass through few segments of networks, which can possibly induce successive delays with different properties due to the variable network transmission conditions which may cause time delay with some different characteristics in practical applications. Based on this, a new model for neural networks with two additive time-varying delays has been proposed in [21,23]. For example, the time delay in the dynamical model such as $\dot{x}(t) = Ax(t) + W_1x(t - d_1(t) - d_2(t))$

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where $d_1(t)$ is the time delay induced from sensor to controller and $d_2(t)$ is the delay induced from controller to the actuator. The stability analysis for such systems has been carried out in [18,20] by using two additive time-varying delay components, $d_1(t) + d_2(t) = d(t)$. Recently, we derived a stability problem for neural networks with two additive time-varying delay components. By constructing the Lyapunov–Krasovskii functional and considering the relationship between time-varying delays and their upper delay bounds, delay-dependent stability criteria are obtained by using reciprocally convex method and convex polyhedron method respectively [21–23].

It is well known that Markovian jump neural networks can be regarded as a special class of systems, which can model dynamic systems whose structures are subject to random abrupt parameter changes resulting from component or interconnection failures, sudden environment changes, etc. (see [24–27]).

Dwell-time switching supervisors force every candidate controller to remain in the loop for at least τ_a of time, thus guaranteeing a fixed dwell-time of τ_0. Unfortunately, with nonlinear system this may lead to finite escape of the closed-loop [28,29]. ADT switching is a class of restricted switching signals that the number of switches in a finite interval is bounded and the average dwell time between consecutive switching is a constant. It is well-known that the ADT scheme characterizes a large class of stable switching signals and its extreme case is the arbitrary switching [30]. Thus, the ADT method is very important not only in theory, but also in practice. Considerable attention has been taken to investigate the stability and stabilization problems both in linear and nonlinear systems (see [31–35] and references there in) applying ADT method.

Meanwhile, the concept of finite-time stability has been revisited in the light of linear matrix inequalities (LMIs) techniques and Lyapunov function theory. Some appealing results were obtained to ensure finite-time stability, finite-time boundedness, and finite-time stabilization of various systems including linear systems, fuzzy systems, network systems, stochastic systems, and so on [36–39]. The problem about finite-time stability, L_2-gain analysis has been widely learned in the literature [40,41]. It is worth pointing out that there is a difference between finite-time stability and Lyapunov asymptotic stability, and they are also independent of each other. Recently, finite-time stability for SNNs based on the technique of ADT and the problem of finite-time boundedness for the SNNs with time delays was investigated (see [42]).

The problem of stability and synchronization for finite-time Markovian jump neural networks with mixed mode-dependent time delays has drawn much attention [43–45], to the best of authors' knowledge no results are available yet on finite-time boundedness, L_2-gain analysis and finite-time control of MJSNNs with additive time-varying delays.

Motivated by this, in this paper finite-time boundedness, finite-time L_2-gain analysis and control of MJSNNs with additive time-varying delays are discussed. Our contributions are as follows: (1) LMI conditions for the system dynamics with the switching signal are given to guarantee finite-time boundedness of MJSNNs; (2) finite-time L_2-gain analysis and control are presented to measure the disturbance tolerance capability of the system within the prescribed time interval by applying ADT method.

The paper is organized as follows. In Section 2, problem formulations and some definitions are presented. In Section 3, based on LMIs, sufficient conditions to guarantee finite-time boundedness and finite-time L_2-gain analysis and control of MJSNNs are given. Finally, a numerical example is presented to illustrate the efficiency of the proposed method in Section 5.

Notation. The notation used in this paper is standard. R^n denotes n dimensional Euclidean, the superscript "T" denotes the transpose and the notation $P > 0$ (≥ 0) means P is real symmetric positive (semi-positive) definite, max(P) and min(P) denote the maximum and minimum eigenvalues of matrix P, respectively. I is an identity matrix with appropriate dimension. diag$\{a_i\}$ denotes the diagonal matrix with the diagonal elements a_i, $(i = 1, 2, \ldots)$. The asterisk $(*)$ in a matrix is used to denote a term that is induced by symmetry. $\mathbb{E}\{\cdot \mid \mathbb{F}(t_k)\}$ stands for the conditional mathematical expectation, where $\mathbb{F}(t_k) = \nu\{(x(t_0), r(t_0)), \ldots, (x(t_k), r(t_k))\}$ is the ν-algebra generated by $\{(x(t_l), r(t_l)),\ 0 \leq l \leq k\}$. The switching sequence is noted as $\{(t_0, \sigma(t_0)), \ldots, (t_k, \sigma(t_k)), \ldots \mid \lim_{k\to\infty} t_k = \infty\}$, where $t_k, \sigma(t_k)$ represent switching instant and switching sequence value.

2. System description and preliminaries

Consider the following MJSNNs with additive time varying delay

$$\left.\begin{aligned} \dot{x}(t) &= -A(r_t,\sigma_t)x(t) + W_0(r_t,\sigma_t)f(x(t)) + W_1(r_t,\sigma_t)f(x(t-d_1(t)-d_2(t))) + B_0(r_t)u(t) + B_1(r_t)w(t),\\ z(t) &= C(r_t,\sigma_t)x(t) + D_0(r_t)u(t) + D_1(r_t)w(t),\\ x(t) &= \vartheta(t), t \in [-d, 0], \end{aligned}\right\} \quad (1)$$

where $t \in \{1, 2, \ldots, N\}$, $N \in \mathbb{N}$, $\mathbb{N}$ is the set of positive integers. $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $z(t) \in \mathbb{R}^r$ is the controlled output, and $w(t) \in \mathbb{R}^s$ is the exogenous disturbance satisfying

$$\|w\|_2^2 = \int_0^T w^T(t)w(t)dt < \delta. \quad (2)$$

$f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T \in \mathbb{R}^n$ is the neuron activation function, $A(r_t, \sigma_t)$ is a positive diagonal matrix, $W_0(r_t,\sigma_t)$, $W_1(r_t,\sigma_t)$, $B_0(r_t)$, $B_1(r_t)$, $C(r_t,\sigma_t)$, $D_0(r_t)$, $D_1(r_t)$ are the weight connection matrices with appropriate