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Nonlinear traveling waves in a two-layer system with heat release/consumption at the interface

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ABSTRACT

The influence of an interfacial heat release and heat consumption on nonlinear convective flows, developed under the joint action of buoyant and thermocapillary effects in a laterally heated two-layer system with periodic boundary conditions, is investigated. Regimes of traveling waves and modulated traveling waves have been obtained. It is found that rather intensive heat sinks at the interface can lead to the change of the direction of the waves' propagation.

1. Introduction

It is known that two-layer liquid systems are subject to numerous instabilities (for a review, see [1,2]). Several classes of instabilities have been found by means of the linear stability theory for purely thermocapillary flows [3–6] and for buoyant-thermocapillary flows [7–10]. For the most typical kind of instability, hydrothermal instability, the appearance of oblique waves moving upstream has been predicted by the theory and justified in experiments [11–13]. However, two-dimensional waves moving downstream have also been observed in experiments [14]. The change of the direction of waves propagation can be caused by the influence of buoyancy [7].

Most of the investigations have been fulfilled for a sole liquid layer with a free surface, i.e., in the framework of the one-layer approach. Recently, Madruga et al. [15,16] studied the linear stability of two superposed horizontal liquid layers bounded by two solid planes and subjected to a horizontal temperature gradient. The analysis has revealed a variety of instability modes. The nonlinear wavy convective regimes in two-layer systems have been described in [17].

In the investigations of convection in two-layer systems, it is typically assumed that the normal components of the heat flux are equal on both sides of the interface. However, there are various physical phenomena which are characterized by a heat release or heat consumption at the interface. Specifically, the interfacial heat release accompanies an interfacial chemical reaction (see, e.g., [18]) and the evaporation [19].

The interfacial heating can be generated, e.g., by an infrared light source. The infrared absorption bands of water and silicone fluids are essentially different [20], therefore the light frequency can be chosen in a way that one of the fluids is transparent, while the characteristic length of the light absorption in another liquid is short.

Another possibility of the interfacial heating may be realized by the use of the ultra-violet radiation: the process of photolysis of hydrogen peroxide H_2O_2 due to the radiation with a wavelength lower than 400 nm, leads to the appearance of OH_radicals, accompanying by the reaction between silicone fluids and OH_radicals with an interfacial heat extraction [21].

It is known that the presence of a constant, spatially uniform heat release or heat consumption at the interface can lead to the appearance of an oscillatory instability [22,23]. Oscillations in [22,23] have been obtained in a two-layer system *heated from below*. Nonlinear convective regimes in a two-layer system filling *a closed cavity* with heat release at the interface have been studied in [24]. The system was heated from the lateral wall.

Let us note that the theoretical predictions obtained for flows in closed cavities cannot be automatically applied for the infinite layers. For the observation of waves in a closed cavity a global instability is needed, while in the case of periodic boundary conditions one observes waves generated by a convective instability of a parallel flow [25]. Also, it should be taken into account, that in the presence of rigid lateral walls the basic flow is not parallel – the lateral walls act as a stationary finite-amplitude perturbation that can produce steady multicellular flow in the part of the cavity and in the whole cavity [25].

In the present paper, the influence of the interfacial heat release and heat consumption on nonlinear convective regimes,

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developed under the joint action of buoyant and thermocapillary effects in a laterally heated two-layer system with *periodic boundary conditions*, has been investigated. Specific regimes of traveling waves and modulated traveling waves have been obtained. It is found that rather intensive heat sinks at the interface can lead to the change of the direction of the waves' propagation.

The paper is organized as follows. In Section 2, the mathematical formulation of the problem in the two-layer system is presented. Numerical simulations of the finite-amplitude convective regimes are considered in Section 3. Section 4 contains some concluding remarks.

2. Formulation of the problem

2.1. Equations and boundary conditions

We consider a system of two horizontal layers of immiscible viscous fluids with different physical properties (see Fig. 1). The variables referring to the top layer are marked by subscript 1, and the variables referring to the bottom layer are marked by subscript 2. The system is bounded from above and from below by two rigid plates, $z = a_1$ and $z = -a_2$. A constant temperature gradient is imposed in the direction of the axis x: $T_1(x, y, a_1, t) = T_2(x, y, -a_2, t) = -Ax + \text{const}$, $A > 0$. A constant heat release of the rate Q_0 (Q_0 may be positive or negative) is set on the interface.

It is assumed that the interfacial tension σ decreases linearly with an increase of the temperature: $\sigma = \sigma_0 - \alpha T$, where $\alpha > 0$.

Let us introduce the following notation:

$$\rho = \rho_1/\rho_2, \quad \nu = \nu_1/\nu_2, \quad \eta = \eta_1/\eta_2, \quad \kappa = \kappa_1/\kappa_2,$$
$$\chi = \chi_1/\chi_2, \quad \beta = \beta_1/\beta_2, \quad a = a_2/a_1.$$

Here $\rho_m, \nu_m, \eta_m, \kappa_m, \chi_m, \beta_m$ and a_m are, respectively, density, kinematic and dynamic viscosity, heat conductivity, thermal diffusivity, thermal expansion coefficient and the thickness of the m-th layer ($m = 1, 2$). As the units of length, time, velocity, pressure and temperature we choose a_1, a_1^2/ν_1, ν_1/a_1, $\rho_1\nu_1^2/a_1^2$ and Aa_1, respectively.

The nonlinear equations of convection in the framework of the Boussinesq approximation for both fluids have the following form (see [1]):

$$\frac{\partial \vec{v}_m}{\partial t} + (\vec{v}_m \cdot \nabla)\vec{v}_m = -e_m \nabla p_m + c_m \nabla^2 \vec{v}_m + b_m G T_m \vec{\gamma},$$
$$\frac{\partial T_m}{\partial t} + \vec{v}_m \cdot \nabla T_m = \frac{d_m}{P}\nabla^2 T_m,$$

$$\nabla \cdot \vec{v}_m = 0. \tag{1}$$

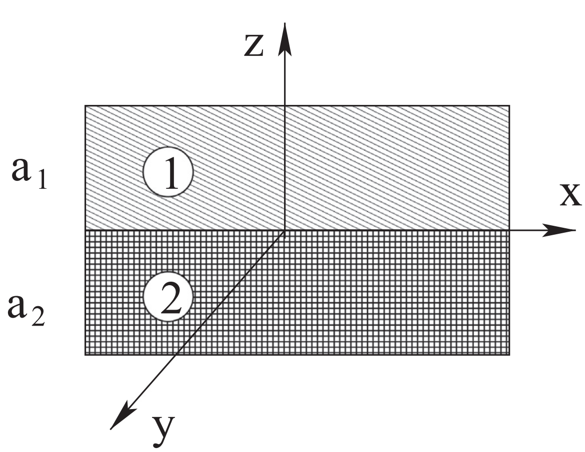

Fig. 1. Geometrical configuration of the two-layer system and coordinate axes.

Here, $\vec{v}_m = (v_{mx}, v_{my}, v_{mz})$ is the velocity vector, T_m is the temperature and p_m is the pressure in the m-th fluid; $\vec{\gamma}$ is the unit vector directed upwards; $b_1 = c_1 = d_1 = e_1 = 1$; $b_2 = 1/\beta$, $c_2 = 1/\nu$, $d_2 = 1/\chi$, $e_2 = \rho$; $G = g\beta_1 A a_1^4/\nu_1^2$ is the Grashof number and $P = \nu_1/\chi_1$ is the Prandtl number for the liquid in layer 1. The conditions on the rigid horizontal boundaries are:

$$z = 1: \quad \vec{v}_1 = 0; \quad T_1 = T_0 - x, \tag{2}$$

$$z = -a: \quad \vec{v}_2 = 0; \quad T_2 = T_0 - x, \tag{3}$$

where T_0 is constant.

We assume that the interface is flat, and it is located at $z=0$.

The boundary conditions on the interface include relations for the tangential stresses:

$$z = 0: \quad \eta\frac{\partial v_{1x}}{\partial z} = \frac{\partial v_{2x}}{\partial z} + \frac{\eta M}{P}\frac{\partial T_1}{\partial x}, \quad \eta\frac{\partial v_{1y}}{\partial z} = \frac{\partial v_{2y}}{\partial z} + \frac{\eta M}{P}\frac{\partial T_2}{\partial y}; \tag{4}$$

the continuity of the velocity field:

$$\vec{v}_1 = \vec{v}_2; \tag{5}$$

the continuity of the temperature field:

$$T_1 = T_2; \tag{6}$$

and the continuity of the heat flux normal components:

$$\kappa\frac{\partial T_1}{\partial z} - \frac{\partial T_2}{\partial z} = -\kappa\frac{G_Q}{G}. \tag{7}$$

Here $M = \alpha\theta a_1/\eta_1\chi_1$ is the Marangoni number, which is the basic non-dimensional parameter characterizing the thermocapillary effect, and $G_Q = g\beta_1 Q_0 a_1^4/\nu_1^2\kappa_1$ is the modified Grashof number determined by the interfacial heat release. Let us note that in [17] $G_Q = 0$.

The boundary-value problem (1)–(7) contains nine thermophysical ($M, G, P, G_Q, \nu, \eta, \kappa, \chi, \beta$) and two geometrical ($a, L$) non-dimensional parameters, where $L = l/a_1$.

2.2. Nonlinear approach

In order to investigate the flow regimes generated by the convective instabilities, we perform nonlinear simulations of two-dimensional flows ($v_{my} = 0$ ($m = 1, 2$); the fields of physical variables do not depend on y). In this case, we can introduce the stream function ψ

$$v_{mx} = \frac{\partial \psi_m}{\partial z}, \quad v_{mz} = -\frac{\partial \psi_m}{\partial x}, \quad (m = 1, 2).$$

Eliminating the pressure and defining the vorticity

$$\phi_m = \frac{\partial v_{mz}}{\partial x} - \frac{\partial v_{mx}}{\partial z},$$

we can rewrite the boundary value problem (1)–(7) in terms of variables ϕ_m, ψ_m, and T_m (see [1]). The calculations have been performed in a finite region $0 \le x \le L$, $-a \le z \le 1$ with periodic boundary conditions on the lateral walls:

$$\psi_m(x + L, z) = \psi_m(x, z); \quad \phi_m(x + L, z) = \phi_m(x, z); \quad T_m(x + L, z) = T_m(x, z) - L;$$
$$m = 1, 2. \tag{8}$$

The boundary value problem is integrated in time with some initial conditions for ψ_m and T_m ($m = 1, 2$) by means of a finite-difference method. Equations and boundary conditions are approximated on a uniform mesh using a second order approximation for the spatial coordinates. The nonlinear equations are solved