Acta Astronautica 128 (2016) 534–539

Contents lists available at ScienceDirect

Acta Astronautica

journal homepage: www.elsevier.com/locate/aa

Angular motion equations for a satellite with hinged flexible solar panel

M.Yu. Ovchinnikov, S.S. Tkachev, D.S. Roldugin*, A.B. Nuralieva, Y.V. Mashtakov

Keldysh Institute of Applied Mathematics, Russian Federation

ARTICLE INFO

Article history:
Received 28 September 2015
Accepted 22 July 2016
Available online 3 August 2016

Keywords:
Non-linear model
Flexible element
Normal mode
Hinged solar panel
Finite element method
FEM

ABSTRACT

Non-linear mathematical model for the satellite with hinged flexible solar panel is presented. Normal modes of flexible elements are used for motion description. Motion equations are derived using virtual work principle. A comparison of normal modes calculation between finite element method and developed model is presented.

1. Introduction

Satellites with large flexible elements (solar panels, antennae, booms etc.) are widely used nowadays. Oscillations of these elements can drastically affect satellite angular motion especially when active attitude control system is used to perform fast angular maneuvers [1,2]. Situation becomes even more complicated if flexible element may rotate under control with respect to the body of the satellite.

For development, study and on-board implementation of the attitude control and identification algorithms the mathematical model should be developed. Two approaches exist to achieve this goal. First, the system of partial and ordinary differential equations (ODEs) can be used [3]. Flexible deformations are described by the partial differential equations and angular dynamics by the ODEs. This approach provides precise results but it is difficult to adapt it for on-board calculation. Second approach utilizes only the system of ODEs. Normal modes are used for flexible deformations description [4–6]. Equations could be derived using either Lagrangian [6] or virtual work [4] approaches. These equations can be easily adapted for the on-board computing.

Normal modes are usually determined by the finite element method (FEM) implementation to the entire satellite. In case of moving parts (e.g. rotating flexible solar panel) normal modes of the system are variable. However, one can use normal modes for the flexible element only and include them in the mathematical model. The paper considers this approach.

* Corresponding author.
E-mail address: roldugids@gmail.com (D.S. Roldugin).

http://dx.doi.org/10.1016/j.actaastro.2016.07.038
0094-5765/

2. Problem definition

Satellite with a rigid body (S) and rotating flexible solar panel (P) is considered (see Fig. 1). Solar panel is attached to the satellite body by, as an example, 1DOF frictionless hinge. A set of reference frames is introduced:

$OXYZ$ is the non-rotating (practically inertial) frame with the origin at Earth's center of mass, OZ axis is perpendicular to the equatorial plane, OX is directed to the vernal equinox.

$O_s xyz$ is the body-fixed frame, its origin lies in the satellite body (S) center of mass and axes are satellite body principal axes of inertia.

$O_p x_p y_p z_p$ is the panel-fixed frame with the origin in the attachment point P, axes are the undeformed panel's principle axes of inertia.

$O_p \xi\eta\zeta$ is the hinge-fixed frame, $O_p\xi$ is the hinge axis of rotation, other axes are chosen arbitrary but perpendicular to the first one.

Arbitrary points' positions of the satellite body and panel are defined as follows (Fig. 1)

$$\mathbf{R}_{si} = \mathbf{R}_s + \mathbf{r}_{si},$$

$$\mathbf{R}_{pi} = \mathbf{R}_p + \mathbf{r}_{pi} + \mathbf{u}_{pi}$$

where $\mathbf{R}_s$ is the radius vector of S center of mass, $\mathbf{R}_p$ is the one of the hinge (hinge is considered as a massless point) with respect to $OXYZ$; $\mathbf{r}_{si}$, $\mathbf{r}_{pi}$ are the radius vectors of body and panel i-th point w.r.t. $O_s xyz$ and $O_p x_p y_p z_p$; $\mathbf{u}_{pi}$ is displacement of panel's i-th point due to deformation.

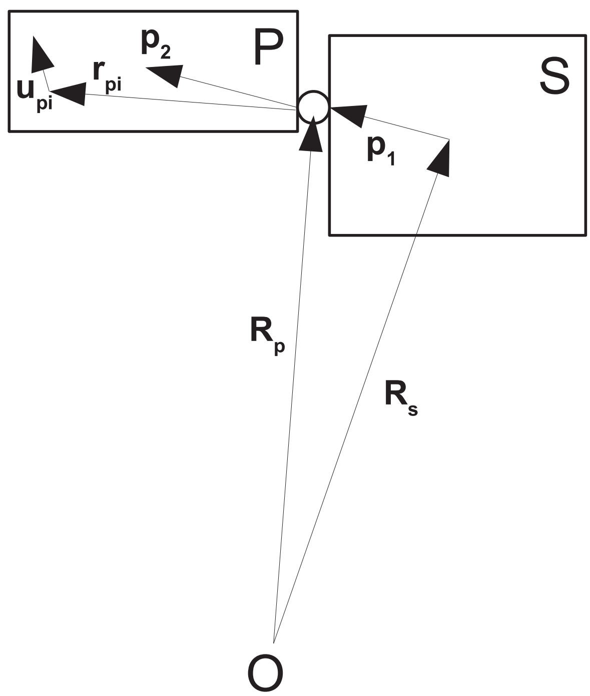

Fig. 1. Satellite with panel scheme.

Vector $\mathbf{p}_1$ in Fig. 1 is the radius vector from S center of mass to the hinge point, vector $\mathbf{p}_2$ links the hinge point and the center of mass of P.

Normal modes are used for deformations definition. Point deformation displacement $\mathbf{u}_{pi}$ is determined as [5–7]

$$\mathbf{u}_{pi} = \mathbf{A}_{pi}(\mathbf{r}_i)\mathbf{q}(t) = \mathbf{A}_{pi}\mathbf{q}.$$

Here $\mathbf{A}_{pi}$ is the 3 by n matrix where n is the number of modes taken into account. Vector $\mathbf{q}$ is the amplitude vector of normal modes. The force due to deformation is determined as [5,7] (no dissipation in oscillations)

$$\mathbf{L}_{pi} = -m_i\mathbf{A}_{pi}\mathbf{\Omega}\mathbf{q}.$$

Here $\mathbf{\Omega} = diag\left(\Omega_1^2, \Omega_2^2, ...\right)$, Ω_j are the normal modes frequencies. This force will be used in equations of motion derivation.

3. Equations of motion

The equations are derived from the virtual work principle

$$\sum_i \left(m_i\ddot{\mathbf{R}}_i - \mathbf{F}_i\right)\delta\mathbf{R}_i = 0 \tag{1}$$

where m_i is the mass of the i-th point of the satellite, $\mathbf{F}_i$ is the force acting upon it and $\delta\mathbf{R}_i$ is the virtual displacement. Sum in (1) is taken over both satellite body and solar panel. When this sum is separated for each element of the satellite it is rewritten as follows

$$\sum_i \left(m_{si}\ddot{\mathbf{R}}_{si} - \mathbf{F}_{si}\right)\delta\mathbf{R}_{si} + \sum_i \left(m_{pi}\ddot{\mathbf{R}}_{pi} - \mathbf{F}_{pi}\right)\delta\mathbf{R}_{pi} - \sum_i \mathbf{F}_{hi}\delta\mathbf{R}_{hi} = 0. \tag{2}$$

The last term in (2) is the total torque in the hinge

$$\sum_i \mathbf{F}_{hi}\delta\mathbf{R}_{hi} = T_e\delta\varphi$$

where $\delta\varphi$ is a virtual rotation and T_e is the total torque in the hinge.

Virtual displacements $\delta\mathbf{R}_{si}$ and $\delta\mathbf{R}_{pi}$ are

$$\delta\mathbf{R}_{si} = \delta\mathbf{R}_s + \delta\mathbf{\theta} \times \mathbf{r}_{si},$$

$$\delta\mathbf{R}_{pi} = \delta\mathbf{R}_s + \delta\mathbf{\theta} \times \left(\mathbf{p}_1 + \mathbf{r}_{pi} + \mathbf{u}_{pi}\right) + \mathbf{e}\delta\varphi \times \left(\mathbf{r}_{pi} + \mathbf{u}_{pi}\right) + \mathbf{A}_{pi}\delta\mathbf{q}_p.$$

Here $\mathbf{e}$ is the hinge axis of rotation. Taking into account that $\delta\mathbf{R}_s$, $\delta\mathbf{\theta}$, $\delta\varphi$, $\delta\mathbf{q}_p$ are independent, Eq. (2) can be rewritten as

$$\begin{gathered}
\sum_i \left(m_{si}\ddot{\mathbf{R}}_{si} - \mathbf{F}_{si}\right) + \sum_i \left(m_{pi}\ddot{\mathbf{R}}_{pi} - \mathbf{F}_{pi}\right) = 0, \\
\sum_i \mathbf{r}_{si} \times \left(m_{si}\ddot{\mathbf{R}}_{si} - \mathbf{F}_{si}\right) + \sum_i \left(\mathbf{p}_1 + \mathbf{r}_{pi} + \mathbf{u}_{pi}\right) \times \left(m_{pi}\ddot{\mathbf{R}}_{pi} - \mathbf{F}_{pi}\right) = 0, \\
\sum_i \mathbf{e}^T\left(\left(\mathbf{r}_{pi} + \mathbf{u}_{pi}\right) \times \left(m_{pi}\ddot{\mathbf{R}}_{pi} - \mathbf{F}_{pi}\right)\right) = M_e, \\
\sum_i \mathbf{A}_{pi}^T\left(m_{pi}\ddot{\mathbf{R}}_{pi} - \mathbf{F}_{pi}\right) = 0.
\end{gathered} \tag{3}$$

This system should be supplemented with kinematic relations for angular motion. Translational, rotational and flexible oscillation motions are fully described. First equation of (3) can be rewritten as

$$m_s\ddot{\mathbf{R}}_s - \mathbf{F}_s + m_p\ddot{\mathbf{R}}_p - \mathbf{F}_p = 0$$

or

$$m\ddot{\mathbf{R}}_0 - \mathbf{F}_0 = 0 \tag{4}$$

It defines translational motion of the center of mass.

After denoting $\mathbf{\omega}$ as the angular velocity of the satellite body and $\mathbf{\omega}_2 = \mathbf{\omega} + \mathbf{e}\psi$ as the total angular velocity of the panel (ψ panel's spin rate) and proper transformations (see Appendix A) Eq. (3) become

$$\begin{aligned}
&\mathbf{J}\dot{\mathbf{\omega}} + \mathbf{S}_{\omega\varphi}\mathbf{e}\dot{\psi} + \mathbf{S}_{\omega p}\ddot{\mathbf{q}}_p + \mathbf{\omega} \times \mathbf{J}\mathbf{\omega} + 2\sum_i \left(\mathbf{r}_{pi} + \mathbf{u}_{pi}\right) \times m_{pi}\mathbf{\omega}_2 \times \dot{\mathbf{u}}_{pi} \\
&\quad + \tilde{\mathbf{J}}_p(\mathbf{\omega} \times \mathbf{e}\psi) + \mathbf{\omega} \times \tilde{\mathbf{J}}_p\mathbf{e}\psi + \mathbf{e}\psi \times \tilde{\mathbf{J}}_p\mathbf{\omega} + \mathbf{e}\psi \times \tilde{\mathbf{J}}_p\mathbf{e}\psi \\
&\quad + m_p\left(\mathbf{p}_1 - \frac{1}{m}\left(m_p\mathbf{p} + m_p\tilde{\mathbf{p}}_2\right)\right) \times \left(2\mathbf{\omega} \times \mathbf{e}\psi \times \left(\mathbf{p}_2 + \tilde{\mathbf{p}}_2\right)\right. \\
&\quad \left. + \mathbf{e}\psi \times \mathbf{e}\psi \times \left(\mathbf{p}_2 + \tilde{\mathbf{p}}_2\right) + 2\mathbf{\omega}_2 \times \dot{\tilde{\mathbf{p}}}_2\right) + \mathbf{f}_p - \mathbf{T}_s = 0,
\end{aligned} \tag{5}$$

$$\begin{aligned}
&\mathbf{e}^T\mathbf{S}_{\omega\varphi}^T\dot{\mathbf{\omega}} + \mathbf{e}^T\mathbf{J}_e\mathbf{e}\dot{\psi} + \mathbf{e}^T\mathbf{S}_{\varphi p}\ddot{\mathbf{q}}_p \\
&\quad + \mathbf{e}^T\left\{\mathbf{\omega} \times \tilde{\mathbf{J}}_p\mathbf{\omega} + \left(m_p\mathbf{p}_2 + \sum_i m_{pi}\mathbf{A}_{pi}\mathbf{q}_p\right) \times \mathbf{\omega} \times \mathbf{\omega} \times \mathbf{p}_1\right. \\
&\quad - \frac{1}{m}\left(m_p\mathbf{p}_2 + \sum_i m_{pi}\mathbf{A}_{pi}\mathbf{q}_p\right) \times \mathbf{\omega} \times \mathbf{\omega} \times \left(m_p\mathbf{p} + \sum_i m_{pi}\mathbf{A}_{pi}\mathbf{q}_p\right) \\
&\quad + \mathbf{e}\psi \times \mathbf{J}_e\mathbf{e}\psi + \tilde{\mathbf{J}}_p(\mathbf{\omega} \times \mathbf{e}\psi) + \mathbf{\omega} \times \tilde{\mathbf{J}}_p\mathbf{e}\psi + \mathbf{e}\psi \times \tilde{\mathbf{J}}_p\mathbf{\omega} \\
&\quad + 2\sum_i \left(\mathbf{r}_{pi} + \mathbf{u}_{pi}\right) \times m_{pi}\left(\mathbf{\omega}_2 \times \dot{\mathbf{u}}_{pi}\right) - \left(m_p\mathbf{p}_2 + \sum_i m_{pi}\mathbf{A}_{pi}\mathbf{q}_p\right) \\
&\quad \times \left(\frac{2}{m}\mathbf{\omega} \times \mathbf{e}\psi \times \left(m_p\mathbf{p}_2 + \sum_i m_{pi}\mathbf{A}_{pi}\mathbf{q}_p\right) + \frac{2}{m}\mathbf{\omega}_2 \times \sum_i m_{pi}\mathbf{A}_{pi}\dot{\mathbf{q}}_p\right) \\
&\quad \left. + \mathbf{f}_e\right\} = M_e,
\end{aligned} \tag{6}$$