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Fully magnetic sliding mode control for acquiring three-axis attitude

M.Yu. Ovchinnikov, D.S. Roldugin *, V.I. Penkov, S.S. Tkachev, Y.V. Mashtakov

Keldysh Institute of Applied Mathematics of RAS, Miusskaya Square, 4, 125047 Moscow, Russia

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ABSTRACT

Satellite equipped with purely magnetic attitude control system is considered. Sliding mode control is used to achieve three-axis satellite attitude. Underactuation problem is solved for transient motion. Necessary attitude is acquired by proper sliding manifold construction. Satellite motion on the manifold is executed with magnetic control system. One manifold construction approach is proposed and discussed. Numerical examples are provided.

1. Introduction

The paper is devoted to three-axis stabilization of satellites using magnetorquers only. The satellite is underactuated since control torque is always perpendicular to the geomagnetic induction vector. This vector rotation allows accessible angular trajectory to be constructed. Sliding control [1,2] is used in this paper to obtain the trajectory mentioned above. Sliding control was already proposed for attitude stabilization of satellites [3–5] including pure magnetic control [6,7]. This scheme was also implemented for underactuated formation flying system [8]. Constant sliding manifold parameters restrict these works application for magnetic attitude control. This paper focuses on acquiring variable manifold parameters. This allows the sliding manifold to change in such a way that satellite angular path may be achieved using magnetorquers only.

2. Problem statement

Three right-handed reference frames are used. Inertial frame $O_aY_1Y_2Y_3$ bases on Earth's axis O_aY_3 and ascending node of Keplerian orbit O_aY_1. Orbital frame $OX_1X_2X_3$ bases on radius-vector of the satellite OX_3 and orbital normal OX_2. Bound reference frame $Ox_1x_2 \times_3$ is tied to the principal axes of inertia of the satellite.

Satellite attitude in inertial or orbital frame is described using Euler equations and kinematic relations based on quaternions and direction cosine matrices. Satellite state vector comprises angular velocity $\boldsymbol{\omega}$ and quaternion $(\mathbf{q}, q_0)$ or direction cosine matrix $\mathbf{A}$ and its components a_{ij}. Dynamical equations of the satellite with inertia tensor $\mathbf{J}$ are written as

$$\mathbf{J}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} = \mathbf{M} + \mathbf{M}_{cntrl} + \mathbf{M}_{dist} \quad (1)$$

where $\boldsymbol{\omega}$ is relative or absolute angular velocity (with respect to orbital or inertial frame), $\mathbf{M}$ is accordingly defined as $\mathbf{M} = \mathbf{M}_{gr} + \mathbf{M}_{rel}$ or $\mathbf{M} = \mathbf{M}_{gr}$ where

$$\mathbf{M}_{rel} = -\mathbf{JWA}\boldsymbol{\omega}_{orb} - \boldsymbol{\omega}_{rel} \times \mathbf{JA}\boldsymbol{\omega}_{orb} - \mathbf{A}\boldsymbol{\omega}_{orb} \times \mathbf{J}(\boldsymbol{\omega}_{rel} + \mathbf{A}\boldsymbol{\omega}_{orb}),$$

$\boldsymbol{\omega}_{orb}$ is the orbital reference frame angular velocity, $\boldsymbol{\omega}_{rel}$ is the satellite angular velocity relative to the orbital frame, $\mathbf{M}_{cntrl} = \mathbf{m} \times \mathbf{B}$ is a control torque, $\mathbf{M}_{gr}$ is the gravitational torque (disturbing torque taken into account by control), $\mathbf{M}_{dist}$ is a disturbance unaccounted in control, $\mathbf{W}$ is a skew-symmetric matrix of angular velocity used for

* Corresponding author. Tel.: +7 926 15 44 983.
E-mail address: roldugindsgmail.com (D.S. Roldugin).

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matrix-based kinematics

$$\dot{\mathbf{A}} = \mathbf{W}\mathbf{A}, \quad \mathbf{W} = \begin{pmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{pmatrix} \tag{2}$$

3. Control construction

Sliding control is designed in two steps. First sliding manifold $\mathbf{x}(\boldsymbol{\omega}, \mathbf{A}, t)$ is constructed in phase space. Satellite desired motion should satisfy $\mathbf{x} = 0$: the satellite moves along the manifold. The manifold is constructed in such a way that attitude $\boldsymbol{\omega} = 0$, $\mathbf{A} = \mathbf{I}$ is asymptotically stable ($\mathbf{I}$ is the identity matrix). Typical sliding manifold for satellite angular motion is

$$\mathbf{x} = \lambda\boldsymbol{\omega} + \boldsymbol{\Lambda}(\boldsymbol{\omega}, \mathbf{S}, t)\mathbf{S} = 0$$

where $\boldsymbol{\Lambda}$ is a positive-defined matrix and λ is a positive value. The scalar function λ characterizes damping control part. It may be substituted with matrix $\boldsymbol{\lambda}$ to allow different gains for each bound frame axis. This may be useful for satellite with particular dynamical configuration, e.g. long cylinder or flat disk. These cases are beyond the scope of this paper. Matrix $\boldsymbol{\Lambda}$ characterizes positional control part. Vector $\mathbf{S}$ characterizes deviation from necessary attitude. It is defined in the same way as the one used in PD-controller [9],

$$\mathbf{S} = \begin{pmatrix} a_{23} - a_{32} & a_{31} - a_{13} & a_{12} - a_{21} \end{pmatrix}^T \tag{3}$$

Control torque should provide motion along the sliding manifold (the second step in control construction). Sliding manifold should be chosen in such a way that control torque is perpendicular to the geomagnetic induction vector. Suppose that control ensures motion along the manifold according to the equation

$$\dot{\mathbf{x}} = -\mathbf{J}^{-1}\mathbf{P}\mathbf{x} \tag{4}$$

where $\mathbf{P}$ is a positive-defined matrix. It represents the time-response of control, e.g. time necessary for the satellite to reach sliding manifold. Taking into account equations of motion rewrite (4) as

$$\dot{\lambda}\mathbf{J}\boldsymbol{\omega} + \lambda\mathbf{J}\dot{\boldsymbol{\omega}} + \mathbf{J}\dot{\boldsymbol{\Lambda}}\mathbf{S} + \mathbf{J}\boldsymbol{\Lambda}\dot{\mathbf{S}} = -\lambda\mathbf{P}\boldsymbol{\omega} - \mathbf{P}\boldsymbol{\Lambda}\mathbf{S}.$$

$\dot{\mathbf{S}}$ is found using (3) and (2). Taking into account dynamical Eq. (1) we obtain

$$\lambda\mathbf{m} \times \mathbf{B} = -\dot{\lambda}\mathbf{J}\boldsymbol{\omega} + \lambda(\boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} - \mathbf{M}) - \dot{\boldsymbol{\Lambda}}\mathbf{J}\mathbf{S} - \boldsymbol{\Lambda}\left(\mathbf{J}\dot{\mathbf{S}} + \mathbf{P}\mathbf{S}\right) - \lambda\mathbf{P}\boldsymbol{\omega} \tag{5}$$

Magnetorquers dipole moment is found from (5). Damping coefficient λ is considered to be known constant. Matrix $\boldsymbol{\Lambda}$ is considered to be symmetric. The problem is to find matrix $\boldsymbol{\Lambda}$ and its derivative. Iterative approach is considered below.

Matrix $\boldsymbol{\Lambda}$ derivative is written as

$$\dot{\boldsymbol{\Lambda}} = (\boldsymbol{\Lambda}(k+1) - \boldsymbol{\Lambda}(k))/\Delta t$$

where Δt is control implementation step. Suppose we know satellite attitude, angular velocity and geomagnetic induction vector for $k+1$ step and previous matrix $\boldsymbol{\Lambda}(k)$. Our purpose is to find $\boldsymbol{\Lambda}(k+1)$. Substituting approximation into (5) we obtain

$$\lambda\Delta t\mathbf{m} \times \mathbf{B} = \left(-\dot{\lambda}\mathbf{J}\boldsymbol{\omega} + \lambda(\boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} - \mathbf{M}) - \boldsymbol{\Lambda}\left(\mathbf{J}\dot{\mathbf{S}} + \mathbf{P}\mathbf{S}\right) - \lambda\mathbf{P}\boldsymbol{\omega}\right)\Delta t - \boldsymbol{\Lambda}(k+1)\mathbf{J}\mathbf{S} + \boldsymbol{\Lambda}\mathbf{J}\mathbf{S} \tag{6}$$

Here all indices except the ones in $\boldsymbol{\Lambda}(k+1)$ are omitted. Introduce notations $\mathbf{a} = \left(-\dot{\lambda}\mathbf{J}\boldsymbol{\omega} + \lambda(\boldsymbol{\omega} \times \mathbf{J}\boldsymbol{\omega} - \mathbf{M}) - \boldsymbol{\Lambda}\left(\mathbf{J}\dot{\mathbf{S}} + \mathbf{P}\mathbf{S}\right) - \lambda\mathbf{P}\boldsymbol{\omega}\right)\Delta t + \boldsymbol{\Lambda}\mathbf{J}\mathbf{S}$, $\mathbf{b} = -\mathbf{J}\mathbf{S}$, $\mathbf{d} = \lambda\Delta t\mathbf{B}$ and rewrite (6) as

$$\mathbf{a} + \boldsymbol{\Lambda}(k+1)\mathbf{b} = \mathbf{m} \times \mathbf{d} \tag{7}$$

Set the new reference frame using basis vectors

$$\mathbf{e}_1 = \mathbf{d}/|\mathbf{d}|, \mathbf{e}_3 = \mathbf{d} \times \mathbf{b}/|\mathbf{d} \times \mathbf{b}|, \mathbf{e}_2 = \mathbf{e}_3 \times \mathbf{e}_1$$

Scalar product of (7) and $\mathbf{d}$ is

$$(\boldsymbol{\Lambda}(k+1)\mathbf{b})\mathbf{d} = -\mathbf{a}\mathbf{d}$$

Taking into account $\mathbf{d} = (d_1, 0, 0)^T$ and $\mathbf{b} = (b_1, b_2, 0)^T$ in the new basis we get

$$\Lambda_{11}(k+1)b_1 + \Lambda_{12}(k+1)b_2 = -a_1 \tag{8}$$

Matrix $\boldsymbol{\Lambda}(k+1)$ construction is performed in a few steps. First $\Lambda_{11}(k+1) > 0$ should be chosen. For example $\Lambda_{11}(k+1) = \Lambda_{11}(k)$. Then using (8)

$$\Lambda_{12}(k+1) = \Lambda_{21}(k+1) = (-a_1 - \Lambda_{11}(k+1)b_1)/b_2$$

Since $\boldsymbol{\Lambda}$ is a positive-defined matrix $\Lambda_{22}(k+1)$ should satisfy

$$\Lambda_{11}(k+1)\Lambda_{22}(k+1) - \Lambda_{12}^2(k+1) > 0 \tag{9}$$

For example $\Lambda_{22}(k+1) = \Lambda_0 + \Lambda_{12}^2(k+1)/\Lambda_{11}(k+1)$, Λ_0 is some constant value. It characterizes positional part contribution in control. However if $\Lambda_{22}(k)$ satisfies (9) the previous step value may be used. Finally $\Lambda_{33}(k+1) = \Lambda_{33}(k)$. Matrix $\boldsymbol{\Lambda}(k+1)$ is then transformed to the bound frame. Expression (7) is used to find control torque and dipole moment. First step initialization is $\boldsymbol{\Lambda}(k+1) = \boldsymbol{\Lambda}(k) = \Lambda_0\mathbf{I}$.

Proposed control cannot be used in the neighborhood of necessary attitude since b_1 and b_2 are both close to zero. To mitigate this problem element $\Lambda_{12}(k+1)$ is constructed according to

$$\Lambda_{12}(k+1) = -(a_1 + \Lambda_{11}(k+1)b_1)/(b_2 + \delta b_2)$$

where δb_2 is a small positive constant. This artificial error leads to slight discrepancy between control torque direction and plane perpendicular to the geomagnetic induction vector. Control torque is projected on this plane to construct dipole moment. Sliding control is designed to acquire, not to maintain necessary attitude. The approach itself is aimed at "sliding" to the vicinity of necessary attitude. It cannot handle disturbance term along geomagnetic induction vector effectively and also suffers from artificial error due to δb_2.

4. Numerical examples

Control implementation involves choosing coefficient λ, positional control contribution Λ_0 and matrix $\mathbf{P}$. These values depend on expected magnetorquers dipole moment value and relation between damping and positional control parts.