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Orbit determination across unknown maneuvers using the essential Thrust-Fourier-Coefficients ☆

Hyun Chul Ko *, Daniel J. Scheeres

The University of Colorado at Boulder, 429 UCB, Boulder, CO 80309-0429, United States

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ABSTRACT

Any maneuver performed by a satellite transitioning between two arbitrary orbital states can be represented as an equivalent maneuver involving Thrust-Fourier-Coefficients (TFCs). With a selected TFC set as a basis, a thrust acceleration can be constructed to interpolate two unconnected states across an unknown maneuver. This representation technique with TFCs enables us to facilitate the analytical propagation of uncertainties of the satellite state. This approach allows for the usage of existing pre-maneuver orbit estimation to compute the orbit solution after the unknown maneuver. In this paper, we applied this approach to orbit determination (OD) problems across unknown maneuvers by appending different combinations of TFCs to the state vector in the batch filter. The aim is to investigate how different maneuver representations with different TFC sets affect the OD solution across unknown maneuvers. Simulation results show that each TFC set provides different representations of the unknown perturbing acceleration, which yields varying magnitudes of delta velocity for a given maneuver. However, OD solutions across unknown maneuvers using different TFC sets display equivalent performance over the post-maneuver arc as long as those TFC sets are capable of generating the apparent secular motion caused by a given unknown maneuver.

1. Introduction

Tracking satellites and predicting their future trajectories is an important subject in space operations. The problem is to propagate the spacecraft orbit state to a future time along with a suitable estimate of its confidence boundary. When satellites perform non-announced maneuvers, post-maneuver analysis tries to reconstruct the orbit trajectory resulting from the maneuver by using orbit determination (OD) results. Without compensating for unmodeled maneuvers, the reconstructed post-maneuver orbit will have inaccurate state predictions and confidence regions. This inaccurate OD solution can be improved by modifying orbital dynamics, or using additional measurements. However, when maneuver dynamics are unknown and a limited number of measurements are available only after a maneuver, a pre-maneuver OD estimation must be incorporated with the post-maneuver measurements to obtain reliable OD solutions across the unknown maneuver.

Accounting for an unknown perturbation is necessary before linking a pre-maneuver orbit estimation with a post-maneuver observation across the unknown maneuver. The common technique operationally used to compensate unknown maneuvers is to add process noises to an OD filter (e.g., State Noise Compensation [1], Dynamic Model Compensation [1], optimal sequential filter with smoother [2]). This allows unknown maneuvers to be estimated based on given information and tracking data. This approach is usually a time-consuming process to

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* Corresponding author.
E-mail addresses: hyun.ko@colorado.edu (H.C. Ko), scheeres@colorado.edu (D.J. Scheeres).

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find valid parameters for obtaining a converged solution. It also requires observation throughout the unknown maneuver period, which often cannot be achieved in data-starved space surveillance environments. To deal with unknown maneuvers in these environments, new approaches have been introduced to detect and reconstruct unknown accelerations by using minimum fuel cost functions [3], or applying optimal control performance metrics [4,5]. This optimal control based approach assumes that the event is fuel optimal, which may not be true for unknown maneuvers.

Recently, a new approach to model unknown perturbations based only on orbit state changes was introduced. Hudson and Scheeres used Fourier series representation of thrust components in order to evaluate the orbit state changes due to dynamic perturbations using the averaged Gauss equations [6]. Ko and Scheeres adapted this approach to efficiently represent the secular effect of any perturbation with essential Thrust-Fourier-Coefficients (TFCs) by analyzing relationships between TFCs and time rates changes of orbital elements [7]. Ko and Scheeres also showed that the chosen essential TFCs can be rigorously estimated as part of an augmented state in the batch filter in order to tie together two disparate states [8].

This paper begins with a review of finding the essential TFCs and briefly explains how to represent an unknown maneuver with different combinations of TFCs. The subsequent section applies the TFC representation technique to the OD process across unknown maneuvers using different essential TFC sets. Then, different types of unknown maneuvers are simulated and a batch filter with each different TFC set is implemented to compare their maneuver representations and OD solutions.

2. Unknown maneuver representation with Thrust-Fourier-Coefficients

It has been shown that the TFC representation of perturbing acceleration can be used to interpolate any separate states and provide a required acceleration to generate an apparent secular behavior [7]. This unknown maneuver representation with TFCs is a mathematical model whose purpose is to rigorously represent the perturbing motion of a satellite under unknown accelerations. When a spacecraft is exposed to an unknown maneuver, the perturbing acceleration, $\vec{U}$, can be represented as a Fourier series expansion in eccentric anomaly (E) along three orthogonal directions (radial, R; along-track, S; cross-track, W) [9]:

$$\vec{U} = U_R\hat{\mathbf{r}} + U_S\hat{\mathbf{s}} + U_W\hat{\mathbf{w}} \tag{1}$$

$$\begin{aligned} U_R &= \sum_{k=0}^{\infty}\left[\alpha_k^R \cos kE + \beta_k^R \sin kE\right] \\ U_S &= \sum_{k=0}^{\infty}\left[\alpha_k^S \cos kE + \beta_k^S \sin kE\right] \\ U_W &= \sum_{k=0}^{\infty}\left[\alpha_k^W \cos kE + \beta_k^W \sin kE\right] \end{aligned} \tag{2}$$

in which α_k and β_k are named Thrust-Fourier-Coefficients (TFCs). Substituting the Fourier series representation of perturbing acceleration components into the Gauss equations, the averaged dynamics equations were found to be a function of only 14 essential TFCs [6]:

$$\vec{c}_{14} = [\alpha_0^R \alpha_1^R \alpha_2^R \beta_1^R \alpha_0^S \alpha_1^S \alpha_2^S \beta_1^S \beta_2^S \alpha_0^W \alpha_1^W \alpha_2^W \beta_1^W \beta_2^W]^T$$

With these 14 essential TFCs, the profile of any unknown thrust can be represented as follows:

$$\begin{aligned} \vec{U} = &(\alpha_0^R + \alpha_1^R \cos E + \beta_1^R \sin E + \alpha_2^R \cos 2E)\hat{\mathbf{r}} \\ &+ (\alpha_0^S + \alpha_1^S \cos E + \beta_1^S \sin E + \alpha_2^S \cos 2E + \beta_2^S \sin 2E)\hat{\mathbf{s}} \\ &+ (\alpha_0^W + \alpha_1^W \cos E + \beta_1^W \sin E + \alpha_2^W \cos 2E + \beta_2^W \sin 2E)\hat{\mathbf{w}} \end{aligned} \tag{3}$$

To achieve a given orbital transfer in the 6-dimensional orbit space, several efficient sets of 6 TFCs were selected to make a fast assessment of any perturbing acceleration [7]:

$$\begin{aligned} \vec{c}_{ess1} &= \left[\alpha_0^R \quad \alpha_0^S \quad \alpha_1^S \quad \beta_1^S \quad \alpha_1^W \quad \beta_1^W\right]^T \\ \vec{c}_{ess2} &= \left[\alpha_0^R \quad \beta_1^R \quad \alpha_0^S \quad \beta_1^S \quad \alpha_1^W \quad \beta_1^W\right]^T \\ \vec{c}_{ess3} &= \left[\alpha_0^R \quad \alpha_1^R \quad \alpha_0^S \quad \alpha_1^S \quad \alpha_1^W \quad \beta_1^W\right]^T \\ \vec{c}_{ess4} &= \left[\alpha_0^R \quad \alpha_1^R \quad \beta_1^R \quad \alpha_0^S \quad \alpha_1^W \quad \beta_1^W\right]^T \\ \vec{c}_{ess5} &= \left[\alpha_1^R \quad \alpha_0^S \quad \alpha_1^S \quad \beta_1^S \quad \alpha_1^W \quad \beta_1^W\right]^T \\ \vec{c}_{ess6} &= \left[\alpha_1^R \quad \beta_1^R \quad \alpha_0^S \quad \beta_1^S \quad \alpha_1^W \quad \beta_1^W\right]^T \end{aligned} \tag{4}$$

Like the full set of 14 TFCs ($\vec{c}_{14}$), these different combinations of 6 TFCs can represent unknown perturbations and provide an analytical solution of control profile to connect two separated states across unknown maneuvers. For example, an unknown perturbing acceleration can be represented by using the 6th essential TFC set ($\vec{c}_{ess6}$):

$$\begin{aligned} \vec{U} = &(\alpha_1^R \cos E + \beta_1^R \sin E)\hat{\mathbf{r}} + (\alpha_0^S + \beta_1^S \sin E)\hat{\mathbf{s}} \\ &+ (\alpha_1^W \cos E + \beta_1^W \sin E)\hat{\mathbf{w}} \end{aligned} \tag{5}$$

There are different ways to compute these TFC values for a given maneuver [7], and those computed TFC values will be different for each TFC set. However, all of them will generate the same secular behavior for a given unknown maneuver as long as they tie two separate states together.

3. Orbit determination across unknown maneuvers

This section briefly explains how the modified batch filter with the essential TFCs works to connect the post-maneuver tracking data to the pre-maneuver orbit state. Without using the pre-maneuver orbit information, achieving reliable post-maneuver state estimation given a small number of post-maneuver measurements is a difficult problem. A conventional method used at the time of encountering an unknown maneuver is to initiate the batch filter over a post-maneuver tracking arc with large initial uncertainties [10]. To distinguish this batch filter from the modified batch filter in this paper, we label it as a regular batch filter that estimates only post-maneuver satellite position and velocity vectors. With the regular