Aerospace Science and Technology 58 (2016) 150–155

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Effect of X-ray energy band on the X-ray pulsar based navigation

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ARTICLE INFO

Article history:
Received 1 February 2016
Received in revised form 18 May 2016
Accepted 23 July 2016
Available online 11 August 2016

Keywords:
X-ray pulsar-based navigation
X-ray energy band
CRLB
RXTE observations

ABSTRACT

The X-ray energy band, which has a direct impact on the detector configuration and the estimation accuracy of the pulse time of arrival (TOA), is an important physical parameter for X-ray pulsar based navigation (XPNAV). Despite nearly five decades of work, there is still no convincing quality analysis for the effect of the X-ray energy on the XPNAV. In this paper, the impacts of the X-ray energy on the XPNAV are quantified for the first time through the Cramér-Rao lower bound (CRLB) theory. A lower bound on the variance of the pulse TOA is derived, and the indicators of the geometric factor and signal to noise ratio (SNR) are presented as the basic selection criteria. The RXTE observations of the Crab pulsar about 11 years (2001–2011) are applied to subdivide the entire 2–60 keV into finer energy windows of 2–10 keV, 10–20 keV, 20–30 keV, 30–40 keV, 40–50 keV and 50–60 keV to obtain pulse profiles in different energy bands. Discrepancies of these pulse profiles are investigated, and their impacts on the navigation accuracy are evaluated. The results demonstrate that the estimation accuracy of the pulse TOA applying the energy band 2–30 keV is improved by 15.54% compared with 2–10 keV, while an improvement of about 15.95% can merely be achieved applying 2–60 keV than 2–10 keV for the Crab pulsar.

1. Introduction

The X-ray pulsar-based navigation (XPNAV), presented in the early 1980s [1], is a promising yet challenging technique in deep-space autonomous navigation [2] and receives relentless attention [3–8]. This developing technique, in a way similar to the position determination method in the global positioning system (GPS), uses the time of arrival (TOA) of the X-ray photons radiated from certain pulsars to compute the position and clock error of the spacecraft [4]. For most pulsars, they cover a broad domain of the X-ray electromagnetic spectrum, ranging from the soft (0.1–20 keV) to hard components (>20 keV) [9,10]. A wide energy band is valuable for the studies of supernovae remnants, black holes and the cosmic background radiation in astronomy and astrophysics, but for an XPNAV system the X-ray energy is dependent on the pulse TOA accuracy and needs a careful analysis and selection to guarantee the estimation accuracy of the pulse TOA [4]. Furthermore, for the designing of X-ray detection instruments, it is also hard to balance the detection capability of a wide spectrum and the detection sensitivity. Previously, the soft X-ray below 10 keV is considered in the XPNAV according to experiences for the high-energy radiation from pulsars is relatively low [4,7]. Most present studies concentrate on the performance improvement of the navigation algorithms, while researches on the effect of the X-ray energy on the navigation precision are relatively scarce and have no convincing quality analysis despite nearly five decades of work [11–16].

The pulse profiles in different energy bands may differ greatly in the background and source arrival rates, and the shape. A comparison of the Crab profiles with energy bands 2–30 keV and 180–580 keV observed respectively by Rossi X-ray Timing Explorer (RXTE) and INTEGRAL instruments shows that there are distinct differences in the intensity and level of bridge emission [10,18]. By dividing the entire energy 2–60 keV into 4 energy windows, the millisecond pulsar PSR J0218+4232, with a period of 2.3 ms, shows significant differences in the intensity and shapes [19]. Many observations of the pulsar Vela X-1 reveal that the pulse profiles exhibit both intensity and energy dependence with a transition from a 5-peaked structure below 5 keV to an asymmetric double-peaked structure above 15 keV [20,21]. Some studies have confirmed that there exists a considerable misalignment in absolute phase of the main pulse measured in the radio, optical, soft and hard X-ray wavelengths [9,22–24]. Due to low photon intensity in the high-energy band, the studies mentioned above have to use a broad energy window, and the influence of the energy on the XPNAV is not involved.

This paper introduces an approach to evaluate the relationship between the estimation accuracy of the pulse TOA and the X-ray energy. In section 2, the mathematical model describing the X-ray pulsar signals is employed and the CRLB of the pulse TOA is

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http://dx.doi.org/10.1016/j.ast.2016.07.017
1270-9638/

derived. In section 3, by using 11-year monitoring data of the Proportional Counter Array (PCA, 2–60 keV) onboard RXTE, the whole energy range is carefully subdivided, and experiments are then undertaken to survey the effect of the X-ray energy on the XPNAV. In section 4, discussion and conclusion are given for the Crab pulsar by using the proposed approach.

2. Effect of the energies on the estimation accuracy of the pulse TOA

Due to different photon arrival rates and absolute phase within different energies, clear distinctions may be synthetically reflected in the shape of pulse profiles. With the pulse profiles in different energy bands, the relationship between the XPNAV and the X-ray energy can be described by the CRLB of the pulse TOA. A detailed discussion of the CRLB of the pulse TOA is provided in section 2.1, and the observed noise variance in the Cramér-Rao inequality is derived in section 2.2.

2.1. The CRLB of the pulse TOA

The observation data over the last decade has confirmed that the pulse profiles of the rotational pulsars are stable in the phase, intensity and pulse width for a certain energy band [17], the relationship between the standard rate function at the solar system barycenter (SSB) and the observed rate function at the spacecraft can be expressed as

$$g(t) = \lambda_b + \lambda_s h\big(\phi(t,\tau)\big) + \upsilon(t) = s(t;\boldsymbol{\theta}) + \upsilon(t), \tag{1}$$

where $g(t)$ and $s(t;\boldsymbol{\theta})$ are respectively the observed and standard rate functions (in cnts/s) standing for the aggregate rate of all arriving photons from the source and background; t is recorded by the Barycentric Coordinate Time (TCB), and τ is time lag of the standard rate function relative to the observed rate function; $\upsilon(t)$ is referred to as the random additive noise; $h(\phi)$ denotes the normalized pulse profile function, which is non-negative and normalized [25]; λ_s and λ_b are effective source and background photon arrival rates [5], satisfying

$$\begin{aligned} \lambda_s &\equiv p_f R_s A\eta \\ \lambda_b &\equiv \big(R_b + (1-p)R_s\big)A\eta \end{aligned} \tag{2}$$

where A and η are the effective area and the detection efficiency of the X-ray detector; p_f is the pulsed fraction parameter of the source, denoted $p_f \le 1$; R_s and R_b are source and background photon arrival rates per unit area. The background photon arrival rate R_b generally consists of the diffuse X-ray background, cosmic X-ray background and detector noise, etc. [26]. When a detector rocks on a target, another onboard detector will always slightly rock off to suppress and remove the background noises through the background subtraction technique. Thus the estimated parameters can be expressed as $\boldsymbol{\theta} = [\theta_1, \theta_2]^T = [\lambda_s, \tau]^T$.

The observed pulsar rate function is usually obtained by the epoch folding procedure, with the process of collecting all photon TOAs into one period and computing the rate of the accumulated photons. During this process, if the number of photons in each bin is larger than 20, the Poisson noise can be approximated by a Gaussian distribution with zero mean and standard deviation [26].

Take the N items of the discrete sequence $\{g(t_i), i = 1, 2, \dots N\}$ to constitute a Gaussian random vector $\mathbf{g}_N = (g(t_1), g_2(t_2), \dots, g(t_N))^T$. Thus, the joint probability density function (pdf) for $\mathbf{g}_N$ conditioned on $\boldsymbol{\theta}$ is

$$p(\mathbf{g}_N|\boldsymbol{\theta}) = \prod_{k=1}^{N}\left\{\frac{1}{\sqrt{2\pi\sigma_k^2}}\exp\left(-\frac{1}{2\sigma_k^2}\big(g(t_k) - s(t_k)\big)^2\right)\right\}. \tag{3}$$

In this equation σ_k is the root to mean square of the observed noise at the kth expansion coefficient. For a long observation time, the amplitude of σ_k is far less than that of the signal, and can be treated as constant σ. By taking the limit of N tending to ∞, the pdf in (3) can be simplified and its log-likelihood function (LLF) is expressed as

$$\ln p\big(g(t)|\boldsymbol{\theta}\big) = -\frac{N}{2}\ln\lim_{N\to\infty}\big(2\pi\sigma^2\big) - \frac{1}{2\sigma^2}\int_0^T\big[g(t) - s(t;\boldsymbol{\theta})\big]^2 dt. \tag{4}$$

The partial derivative of the LLF with respect to the unknown parameter θ_i is

$$\frac{\partial \ln p(g(t)|\boldsymbol{\theta})}{\partial\theta_i} = \frac{1}{\sigma^2}\int_0^T\big[g(t) - s(t;\boldsymbol{\theta})\big]\frac{\partial s(t;\boldsymbol{\theta})}{\partial\theta_i}dt. \tag{5}$$

The elements of the Fisher information matrix (FIM) are given by computing the expectation of the second partial derivative of the LLF

$$\mathbf{J}_{ij}(\boldsymbol{\theta}) = -E\left[\frac{\partial^2 \ln p(g(t)|\boldsymbol{\theta})}{\partial\theta_i\partial\theta_j}\right]. \tag{6}$$

Submitting (1) into (6), the elements of FIM are obtained

$$\mathbf{J}(\boldsymbol{\theta}) = \begin{bmatrix} \int_0^T h(t,\tau)^2 dt & \frac{b}{\sigma^2}\int_0^T h(t,\tau)\frac{\partial h(t,\tau)}{\partial\tau}dt \\ \frac{b}{\sigma^2}\int_0^T h(t,\tau)\frac{\partial h(t,\tau)}{\partial\tau}dt & \frac{b^2}{\sigma^2}\int_0^T\left(\frac{\partial h(t,\tau)}{\partial\tau}\right)^2 dt \end{bmatrix}. \tag{7}$$

The CRLB of an unbiased estimator $\boldsymbol{\theta}$ is given by $\mathrm{CRLB}\{\theta_i\} \ge \mathbf{J}_{ii}^{-1}(\boldsymbol{\theta})$, where $\mathbf{J}_{ii}^{-1}(\boldsymbol{\theta})$ is the diagonal elements of the inverse of the FIM, and the CRLB for the estimation τ can be written as

$$\begin{aligned} &\mathrm{var}(\hat{\tau}) \\ &\ge \frac{\sigma^2}{\lambda_s^2}\frac{1}{\int_0^T\left(\frac{\partial h(t,\tau)}{\partial\tau}\right)^2 dt - \left(\int_0^T h(t,\tau)\frac{\partial h(t,\tau)}{\partial\tau}dt\right)^2 / \int_0^T h(t,\tau)^2 dt} \\ &= \frac{\Gamma}{\mathrm{SNR}}. \end{aligned} \tag{8}$$

Note that (8) is the general variance bound of the pulse TOA, which provides a benchmark for the theoretical estimator performance limit. The first item, the signal-to-noise ratio (SNR) in the right side of the inequality, shows the accuracy of the pulse TOA is proportional to the effective source arrival rate λ_s and in inverse proportion to σ, indicating that both a large detection area and long observation time can effectively improve the estimation accuracy. The second item, the geometric factor Γ, is used to describe the effect of the pulse shape on the TOA estimation accuracy. However, the observed noise variance is unknown and needs to be estimated in (8).

2.2. Estimation of the observed noise variance

To derive the noise variance, an important aspect is to carefully analyze the statistical characteristics of the observed noise. Suppose a total of M photon events are recorded during the whole observation time. Let T_{obs} be the observation time and t_i the TOA of the ith X-ray photon. The set of $\{t_1, t_2, \cdots, t_M\}$ is a randomly increasing sequence. Assume the pulse cycle T is divided into N_b bins with the equal-length Δt which is generally very small compared with the pulse cycle T. The Poisson random variable $X_{i,j}$ means the number of photons in the jth bin of the ith pulse cycle and its expression is given by