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Nonlinear buckling analysis of the conical and cylindrical shells using the SGL strain based reduced order model and the PHC method

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ABSTRACT

Thin-walled conical and cylindrical shells subjected to axial compression often show a snap-back response in the presence of buckling. Newton iterations based path-following methods cannot trace reliably the snap-back response due to the extremely sharp turning angle near the limit point, and the original Koiter–Newton method also meets difficulties to achieve a complete post-buckling response beyond the limit point. In this paper, the improved Koiter–Newton method is proposed to trace the post-buckling path of cylinders and cones, in the framework of the reduced-order modeling technique. The polynomial homotopy continuation (PHC) method is used to solve the lower-order nonlinear reduced order model reliably and efficiently. The simplified Green–Lagrange (SGL) kinematics which consider the stress redistribution after buckling are implemented into the construction of the reduced order model to produce accurate results for curved shells. The numerical results presented reveal that the improved method is a robust and efficient technology to achieve the entire nonlinear response for the snap-back case.

1. Introduction

Thin-walled conical and cylindrical shells are commonly used as primary components in weight-critical structure engineering, such as aircrafts and rockets, due to the high specific strength and stiffness [1–3]. Their load-carrying capabilities are often determined by the buckling loads which may be much lower than the failure loads of materials. These shell type structures, which exhibit an unstable post-buckling behaviour, are highly sensitive to the initial imperfections, especially to the geometric imperfection [4].

Nonlinear structural analysis based on a path-following technique is commonly used to trace the response curve and to predict the load-carrying capacity of shell structures in the presence of buckling [2,5]. Snap-through and snap-back responses are two main phenomenons usually associated with the buckling of shell structures [6]. Some variants of the classical Newton method, i.e. the arc-length method [7] and norm flow method [8], have been proved to deal with the snap-through case very well. However, the above methods encounter difficulties with a snap-back response of cylindrical shells [9], where extremely sharp turning angles are present [3,5,10]. A significant reduction of the incremental step size is required to distinguish properly the two closely spaced path segments near the limit point [5,11]. When instabilities are localized, there will be a local transfer of strain energy from one part of the model to neighboring parts which may prevent the success of global solution methods. This class of problems must be solved either dynamically or with the aid of artificial damping. Thus, a combination of the displacement control and a damping factor is commonly used to pass the limit point of a snap-back response [5,10].

The Koiter–Newton (KN) method [12,13] has been proposed based on the reduced-order modelling technique to trace the nonlinear equilibrium path in a stepwise manner. In each step, the method combines a prediction phase using a nonlinear reduced order model (ROM) based on Koiter asymptotic analysis [14–16] with a Newton iteration based correction procedure, thus allowing the algorithm to use fairly large step sizes. In the original Koiter–Newton method, the classical arc-length method is used to solve the nonlinear system of equations associated with the reduced order model. Instead of solving a large-scale nonlinear system generated from the full finite element model [17], the lower-order reduced order model can provide a much more reliable solution to pass the extremely sharp turning angle in the snap-back case. However, the path-following performance is still very sensitive to the solution parameters related to the arc-length method and the method meets difficulties to trace the post-buckling path

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that is far beyond the limit point. Actually, the nonlinear algebraic equations, that is the reduced order model, can be expressed as polynomial equations. Thus, the polynomial homotopy continuation (PHC) method [18,19] can be used as a reliable and efficient tool to solve the lower-order reduced order model. The construction of the reduced order model requires derivatives of the strain energy with respect to the degrees of freedom up to the fourth order, which is two orders more than traditionally needed for a Newton based nonlinear finite element technique. The von Kármán kinematics have been used in the finite element implementation of the original Koiter–Newton method [13]. The nonlinear in-plane rotation terms are neglected to facilitate the high order derivatives of the strain energy. However, these terms might be negligible for flat plate situations but they indeed play a major role in structures consisting of assembly of flat plates or curved shells. An alternative way to alleviate the shortcoming is to use the simplified Green–Lagrange (SGL) strain tensor which has been successfully implemented in the former Koiter reduction method [16].

The contribution of this paper distinguishes significantly from previous publications [6,12,13,20] in the improvement of the original Koiter–Newton method to be more applicable for the snap-back case. The lower-order reduced order model is solved using the polynomial homotopy continuation method, to trace reliably the entire snap-back response. The simplified Green–Lagrange kinematics are implemented into the Koiter–Newton method to obtain a more accurate results for the curved shells. We carefully test the improved Koiter–Newton method for the snap-back behaviour of thin-walled cylindrical and conical shells to reveal the performance of the method for extremely sharp turning angles at the limit point. We demonstrate that the method is capable to handle these numerically severe test cases reliably and accurate and thus outperforms most of the state-of-the-art solution methods.

The rest of the paper is organized as follows: a brief introduction of the shell theory and the SGL strain used in this study is given in section 2. The Koiter–Newton method and the PHC method used to solve the reduced order model are presented in section 3. Numerical examples of cylinders and cones used to demonstrate the success of the method are provided in section 4. We summarize the paper and draw conclusions in section 5.

2. Shell theory based on simplified SGL strain

Based on the classical plate theory (Kirchhoff–Love hypothesis), the three displacement components (u, v, w) of a thin plate are expressed as:

$$\begin{cases} u(x,y,z) &= u^0(x,y) - z\dfrac{\partial w}{\partial x} \\ v(x,y,z) &= v^0(x,y) - z\dfrac{\partial w}{\partial y} \\ w(x,y) &= w^0(x,y) \end{cases}, \tag{1}$$

where $u^0(x,y)$, $v^0(x,y)$ and $w^0(x,y)$ are the displacement components related to the mid-plane of the plate.

The total strain vector $\boldsymbol{\varepsilon}$ for the plate is:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_m + z\boldsymbol{\kappa}, \tag{2}$$

where $\boldsymbol{\varepsilon}_m$ and $\boldsymbol{\kappa}$ are the in-plane strain vector of the mid-plane and the curvature vector of the plane.

To consider the geometrical nonlinearities of the plate, the simplified Green–Lagrange strain kinematics are used for the mid-plane strain $\boldsymbol{\varepsilon}_m$, as given by:

$$\boldsymbol{\varepsilon}_m = \left\{ \varepsilon_x \quad \varepsilon_y \quad \varepsilon_{xy} \right\}, \tag{3}$$

where

$$\begin{cases} \varepsilon_x &= \dfrac{\partial u}{\partial x} + \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x}^2 + \dfrac{\partial w}{\partial x}^2\right) \\ \varepsilon_y &= \dfrac{\partial v}{\partial y} + \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y}^2 + \dfrac{\partial w}{\partial y}^2\right) \\ \varepsilon_{xy} &= \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) + \dfrac{1}{2}\left(\dfrac{\partial w}{\partial x}\dfrac{\partial w}{\partial y}\right) \end{cases}, \tag{4}$$

The simplified Green–Lagrange strain kinematics include some nonlinear in-plane rotation terms, which consider the stress redistribution in the post-buckling deformation.

The constitutive relationship of the plate is written as:

$$\begin{Bmatrix} \mathbf{N} \\ \mathbf{M} \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \boldsymbol{\varepsilon}^m \\ \boldsymbol{\kappa} \end{Bmatrix}, \tag{5}$$

where $\mathbf{N}$ and $\mathbf{M}$ are the membrane force vector and the bending moment vector, respectively, and matrices A, B and D are the membrane stiffness, membrane-bending coupling stiffness and bending stiffness, respectively. For the isotropic shell, the coupling stiffness B equals to be 0. For laminated composite plate, the material stiffness A, B and D can be calculated using the classical lamination theory.

3. The improved Koiter–Newton method

In the following we briefly present the basic ideas and principles of the Koiter–Newton method. In particular, we introduce the polynomial homotopy continuation method used to solve the reduced order model. For a detailed description of the theory, we point the reader to work [12,13,18].

3.1. Construction of the reduced order model

The Koiter–Newton method is based on a step by step procedure to trace the equilibrium path of the deforming structure, that is similar to classical path-following techniques [21]. The unique properties and algorithmic differences compared to standard technologies are given in the following with the help of Figs. 1 and 2. In addition, the red contents written in Fig. 1 are the main improvements made in this study, compared to the original KN method.

Starting from a known nominal equilibrium state $(\mathbf{q}_0, \lambda_0)$, usually the un-deformed configuration of a structure, the equilibrium state is represented by a set of nonlinear algebraic equations:

$$\mathbf{f}_{int}(\mathbf{q}) = \lambda \mathbf{f}_{ext} \tag{6}$$

where $\mathbf{f}_{int}$ and $\mathbf{f}_{ext}$ are the internal force vector and external force vector, respectively, and λ is the load parameter, and $\mathbf{q}$ is the displacement vector.

The corresponding reduced order model at this known state is constructed within the framework of the Koiter–Newton method, to be:

$$\bar{\mathcal{L}}(\boldsymbol{\xi}) + \bar{\mathcal{Q}}(\boldsymbol{\xi},\boldsymbol{\xi}) + \bar{\mathcal{C}}(\boldsymbol{\xi},\boldsymbol{\xi},\boldsymbol{\xi}) = \boldsymbol{\phi} \tag{7}$$

where $\bar{\mathcal{L}}$, $\bar{\mathcal{Q}}$ and $\bar{\mathcal{C}}$ are still to be determined linear, quadratic and cubic forms of the approximated load amplitudes $\boldsymbol{\phi}$. These forms can also be represented by a two-dimensional tensor $\bar{\mathsf{L}}$, a three-dimensional tensor $\bar{\mathsf{Q}}$ and a four-dimensional tensor $\bar{\mathsf{C}}$ of order $(1+m)$, respectively, where m is the number of the closely-spaced buckling modes of the structure. $\boldsymbol{\xi}$ are generalized displacements or perturbation parameters.

The construction of the reduced order model (7) is illustrated in Fig. 1. The basic idea involves a modification of Koiter's asymptotic theory to make it applicable already from the unloaded state.