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The modified Brazier approach to predict the collapse load of a stiffened circular composite spar under bending load

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ABSTRACT

The response of a stiffened circular composite spar subjected to bending was studied analytically using a modified Brazier approach. The spar caps were placed in the top and bottom sectors of a spar to increase the bending capabilities such as the rigidity and collapse load. The mathematical model to predict the collapse load of the stiffened circular spar was derived using the linear elastic energy method. Numerical simulations were performed to estimate the effect of the spar caps on the failure load and compared with the presented model. A static experimental test was performed on a prototype stiffened spar, and the test results were compared with a mathematical model. The results show that the modified Brazier approach can reasonably predict the collapse load.

1. Introduction

A high strength-to-weight ratio can make composites the best candidate materials for ultra-light unmanned air vehicles. These days, composite materials have been widely used in aircraft design such as solar powered high altitude long endurance unmanned air vehicles (HALE-UAV) [1]. In this study, a tubular composite tube was applied to the main spar of a HALE-UAV wing. Fig. 1 shows the particular section configuration of the wing. Additionally, Fig. 2 shows the prototype of a typical wing-section. The main carbon/epoxy spar should carry all of the shear/bending/torsion loads applied to the wing structure. When a thin-walled tubular member is subjected to a bending moment, its cross section becomes oval as the curvature is increased.

Fig. 3 shows the radial direction deformation of a circular spar. This instability was first investigated by Brazier [2]. Brazier showed that the nonlinearities are due to the ovalization of the cross-section that occurs under a bending load. This ovalization reduces the cross-section's moment of inertia and the load carrying capacity. Brazier mentioned that the ovalization can cause limit moment instability. Brazier's simple theory was developed for cylindrical shells. The reliability of the approach has been demonstrated by Fabian [3] who used nonlinear shell equations and showed that the bending response obtained from Brazier's simple theory agrees well with the nonlinear solution. Fabian concluded that a bifurcation moment almost coincides with a limit moment when a circular tube is subjected to bending without an axial load. Tatting et al. [4] showed that the local buckling of a cylindrical shell almost occurs before the limit moment is reached. They investigated the local buckling behavior of composite shells showing cross-sectional deformations related to Brazier's flattening effect.

Corona et al. [5] analyzed the response and failure of thin walled composite cylinders in bending. They included a nonlinear prebuckling response and failure of the shell type bifurcation in the formulation. They also included appropriate material failure criteria to indicate failure. Fuchs and Hyer [6] dealt with short, thin-walled laminated composite cylinders under bending. Geometrically nonlinear shell theory was used to investigate the bending response of circular cylinders. They concluded that the laminate stiffness properties of a tubular tube are important in determining the response of short cylinders. The Poisson's ratio also has an important role in radial direction displacement. Fam [7] showed the behavior of cantilevered composite tubular tubes under lateral loads. A three-dimensional finite element model, validated with experimental results, was used to examine the behavior of circular composite poles with various laminates and geometric parameters. The numerical model accounted for geometric nonlinearities and for the material failures of composite laminates and buckling failures. Sadowski [8] studied the adequacy of the shell element on limit moment and bifurcation buckling. The results showed that the using a shell element to model circular tubes under bending is economical as the R/t ratio rises above 25. In this study, a modified Brazier approach was proposed by author to in-

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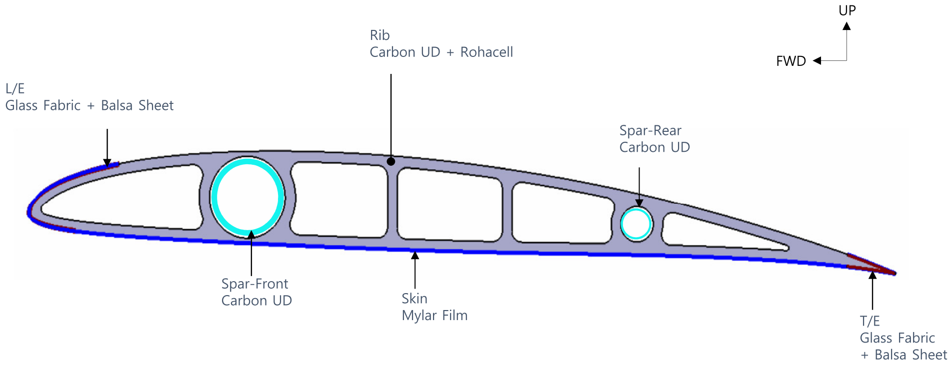

Fig. 1. Wing cross-section.

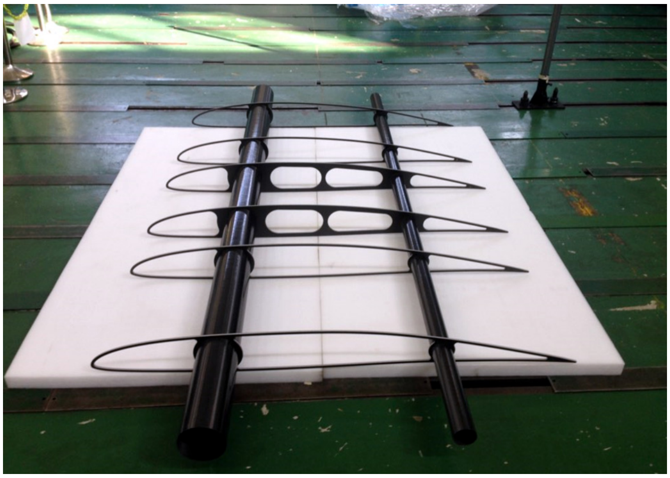

Fig. 2. Prototype of typical wing-section.

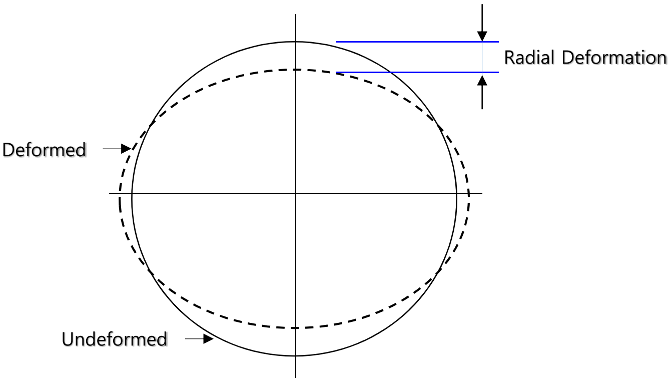

Fig. 3. Cross-section displacement.

vestigate the effect of a spar cap that is reinforced in a longitudinal direction to improve the bending capacity of tabular spars.

2. Mathematical model

The Brazier effect is the nonlinear behavior of long tubes with deformable cross-sections under bending. When a cylindrical tube is very long, the failure mode of cylinders is determined by flattening the cross-section. The Brazier is able to predict the collapse load. The critical bending moment can be estimated as follows:

$$M_{cr} = 2\pi R\sqrt{\frac{8}{27}E_x H D_{22}}, \tag{1}$$

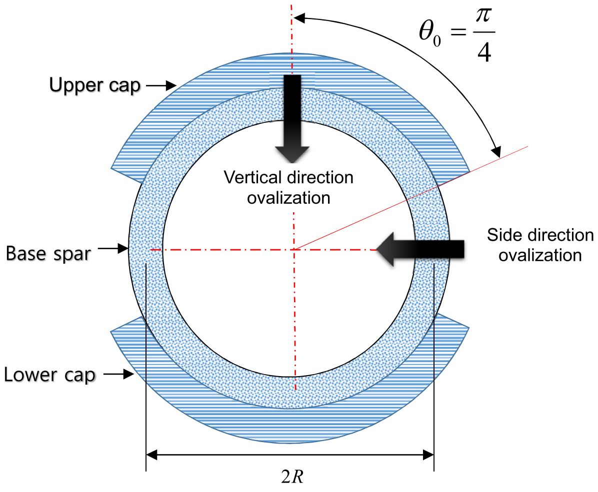

Fig. 4. Cross-section of a circular spar with cap.

where E_x is the axial direction apparent modulus of the laminate, H is the thickness of the laminate; R is the radius of the spar and D_{22} is the bending stiffness in the circumferential direction. However, Eq. (1) cannot be used to determine the critical limit moment of a nonuniform cross-section. Fig. 4 shows the analysis model of a circular spar with a cap. To investigate the physical meaning of Eq. (1), the equation can be rewritten as follows:

$$M_{cr} = \sqrt{\frac{8}{27}}\sqrt{2\pi RHE_x}\sqrt{2\pi RD_{22}}, \tag{2}$$

where $\sqrt{2\pi RHE_x}$ and $\sqrt{2\pi RD_{22}}$ are related to the cross-section axial and bending (ovalization) rigidity of the circular tube. In order to extend the Brazier approach to a nonuniform cross-section, the following equation is proposed to determine the critical bending moment of the circular spar with a cap:

$$M_{\text{mod}} = M_{cr} f_a f_b, \tag{3}$$

where f_a, f_b are the correction factors for the axial and cross-section bending rigidities. These factors are used to consider the effect of the cap on the critical bending moment. f_a is the axial rigidity ratio between spars that have and do not have a cap. It is derived as follows:

$$f_a = \sqrt{\frac{E_c A_c}{E_s A_s}}, \tag{4}$$

where $E_c A_c$ is the axial rigidity of a spar with a cap, and $E_s A_s$ is the axial rigidity of a spar without a cap. The shortening of the diameter in the plane of bending is known as ovalization. f_b is the spar cross-section bending rigidity ratio between spars that have