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Attitude control of an underactuated spacecraft using tube-based MPC approach

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ABSTRACT

Feasibility of achieving 3-axis stabilization of an asymmetric spacecraft for cases where there is no control available in one axis (underactuated spacecraft) is explored in this paper. A novel control design methodology is presented which can steer an underactuated spacecraft to the close neighborhood of the origin. A passive fault tolerant control (FTC) is defined which controls and maintains the attitude of spacecraft near the desired point in presence of uncertainty, disturbances, control constraints and actuator faults. According to the general conditions of an underactuated spacecraft, a tube-based model predictive control (MPC) scheme is developed based on the nonlinear kinematic and dynamic equations of a spacecraft motion. The controller solves two optimal control problems, one which solves a standard problem for the nominal system to define a central guide path, and an ancillary problem to steer the states towards the central path despite the uncertainties and disturbances. Numerical simulation results for a detumbling of a desired orientation maneuver are presented to illustrate the capability of the proposed attitude control method.

1. Introduction

During the past two decades, extensive studies have been carried out on the development of active and passive FTC approaches for spacecraft attitude control. Among the various applications of FTC approaches, actuator faults were more attractive in the field of spacecraft attitude control [1]. In the case of actuator failures, adaptive control techniques can be used to generate three independent control torques using the remaining functional actuators [2]. This could be possible by using a reconfigurable control methodology for a spacecraft equipped with redundant actuators. However, the control of spacecraft becomes more challenging if the fault tolerant control system (FTCS) encounters the underactuated situation which can deliver torque components about two axes only due to a failure of one or more actuators. The problem of attitude control of underactuated spacecraft has been investigated by many researchers and several control algorithms have been proposed [3–9]. Stabilization of underactuated system not only guarantees safe operations of the spacecraft, but also has special significance for small satellites and deep space probes which have weight, size and cost limitation [10].

The investigation on stabilization of underactuated spacecraft attitude kinematics and dynamics began in [11] by Crouch. He presented necessary and sufficient conditions for controllability of the system equipped with gas jets or momentum exchange devices. He concluded that the controllability for a spacecraft is impossible with fewer than three momentum exchange devices, while it is possible for a spacecraft with two independent gas jets. The initial researches only dealt with the stabilization of angular velocities of underactuated rigid spacecraft [7,12–16]. The objective was to null the angular velocities of spacecraft and stabilize it dynamically. However stabilization of the attitude state vector consists of kinematics and dynamics variables and is a much more complicated problem. Byrnes and Isidori in [17] found that while the rigid model of spacecraft cannot be locally asymptotically stabilized by smooth feedback, but nonlinear feedback laws can be derived which control the closed-loop trajectories to a revolute motion about an axis of rotation. Tsiotras et al. in [18–21] proposed a new method for stabilizing the angular velocity and attitude of an axisymmetric rigid spacecraft by definition of a new representation of attitude kinematics. Bajodah in [4,22] addressed the rate-only stabilization problem using singularity perturbed feedback linearization and generalized inverse control methodologies. Hall et al. in [3] developed Bajodah's approach and proposed a controller to stabilize both the kinematics and dynamics of an underactuated asymmetric spacecraft. Although he could stabilize an underactuated spacecraft completely but it is supposed that the model has no external disturbance with no uncertainty and constraint. Godard and Kumar in [8] developed a robust nonlinear

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control scheme in presence of gravity gradient torque and external disturbances for cases where there is no control available on either roll or yaw axis. They involved the gravity gradient in attitude dynamics equations and satisfied controllability of the linearized equations. Hence the settling time for stabilization is as long as orbital period. Wang et al. in [10] proposed a controller for rigid spacecraft with periodical oscillation disturbances using the back stepping control method and internal model principle. In [23,24] Zhuang et al. proved that asymmetric underactuated spacecraft is orbitally flat and he proposed a method for time optimal trajectory generation based on this property. Among all related researches may only Pong in [9] considered the control input constraint in attitude control problem of underactuated asymmetric spacecraft. MPC which is proposed by Pong is stated as a practical approach for stabilizing both translational and attitude dynamics of an underactuated spacecraft. In this approach unactuated axis is modeled by definition of zero equality constraint while two other actuated axes are bounded. However the stability of MPC is not considered in presence of uncertainties and external disturbances.

Majority of the past achievements mainly focus on the stability of a known model of underactuated spacecraft without any constraints on input controls. In this paper, considering the more general condition on the spacecraft dynamics model, a new algorithm is proposed for underactuated spacecraft attitude control based on tube-based MPC which is proposed by Mayne and Falugi et al. in [25]. Tube-based MPC is an implementable form of feedback MPC in which the decision variable in the optimal control problem solved online is a sequence of control laws, rather than a sequence of control actions employed in conventional open-loop MPC [25]. Using tube-based MPC all predictions of the state and control variables are confined to the tighten constraint set, so the real dynamics of controlled systems will never deviate the constraints under the impaction of external disturbances. Hence tube-based MPC is proposed to stabilize the dynamics and kinematics of a spacecraft with uncertainty and disturbances and also bounded control inputs.

The paper is organized as follows. Section 2 introduces the nonlinear model of attitude dynamics and kinematics of the spacecraft. According to the proposed controller, problem formulation is described in Section 3. Control algorithm based on tube-based MPC is formulated with stability conditions for robustness against uncertainties and external disturbances in Section 4. For a detailed assessment of the system performance under the proposed controller, the results of numerical simulations are represented in Section 5. Finally the conclusions of the presented study are stated in Section 6.

2. Spacecraft model

In this section, the attitude dynamics model is established for an underactuated spacecraft with two actuators in the presence of external disturbance and uncertainties in moment of inertia. Finally, the attitude kinematics model is described by the elements of quaternion.

2.1. Attitude dynamics

The dynamics of an asymmetric spacecraft can be described by the well-known Euler's equations. For the general case, where the control axes do not coincide with the principal axes of inertia, the attitude dynamics equation becomes:

$$\dot{\boldsymbol{\omega}} = -J^{-1}\omega^{\times} J\boldsymbol{\omega} + \boldsymbol{u} + \boldsymbol{d}; \tag{1}$$

where, $\boldsymbol{\omega} \in \mathbb{R}^{3\times1}, \forall\boldsymbol{\omega} = [\omega_1, \omega_2, \omega_3]^T$ is angular velocity vector, $J \in \mathbb{R}^{3\times3}$ is the symmetric positive definite moment of inertia of the spacecraft; $\boldsymbol{u} = J^{-1}\boldsymbol{T} \in \mathbb{R}^{3\times1}$ and $\boldsymbol{d} = J^{-1}\boldsymbol{D} \in \mathbb{R}^{3\times1}$ where $\boldsymbol{T} = [T_1, T_2, T_3]^T$ is the control input and $\boldsymbol{D} = [D_!, D_2, D_3]^T$ is the bounded disturbance torque contains external disturbance and another ones which produce by uncertainties such as moment of inertia. $\boldsymbol{D}$ can also include the flexibility of the spacecraft such as [26]. And $\omega^{\times}$ is skew symmetric matrix of $\boldsymbol{\omega}$ that is defined as follows:

$$\omega^{\times} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \tag{2}$$

Without loss of generality and just for simplicity in modeling of an underactuated spacecraft, z axis of the body coordinate system supposed to be unactuated, so $\boldsymbol{u} = [u_1 \ \ u_2 \ \ 0]^T$. For the nominal case where there is no disturbance and uncertainty, dynamics equation can be stated as follows:

$$\dot{\boldsymbol{\omega}}_n = -J_n^{-1}\omega_n^{\times} J_n\boldsymbol{\omega}_n + \boldsymbol{u}; \tag{3}$$

In which $\boldsymbol{\omega}_n$ and J_n are the nominal angular velocity vector and nominal moment of inertia matrix.

2.2. Kinematics model

The kinematic equation through unit quaternion representation is given as:

$$\dot{\boldsymbol{q}} = \frac{1}{2}(q_0 \mathrm{I}_3 + q^{\times})\boldsymbol{\omega}, \tag{4}$$

$$\dot{q}_0 = -\frac{1}{2}\boldsymbol{q}^T\boldsymbol{\omega}, \tag{5}$$

$$\boldsymbol{q}^T\boldsymbol{q} + q_0^2 = 1. \tag{6}$$

In which $\boldsymbol{q} = [q_1, q_2, q_3]^T$ is vector part of the quaternion; q_0 is scalar part of the quaternion; I_3 is identity $[3 \times 3]$ matrix and $q^{\times}$ is the skew symmetric matrix of $\boldsymbol{q}$ similar to (2). In the following, $[q_0, \boldsymbol{q}^{\boldsymbol{T}}] = [1, 0, 0, 0]^T$ is considered as the desired attitude equilibrium point for the controller design. It is also obvious from the kinematic model of a rigid body that $-1 \le q_i \le 1$, $(i = 0, 1, 2, 3)$ accordingly.

3. Problem formulation

The constrained discrete-time nonlinear system is described

$$\boldsymbol{x}^+ = f(\boldsymbol{x}, \boldsymbol{u}) + \boldsymbol{\epsilon} \tag{7}$$

where $\boldsymbol{x} \in \mathbb{R}^{7\times1}, \forall\boldsymbol{x} = [q_0, \boldsymbol{q}^T, \boldsymbol{\omega}^T]^T$ is the state vector, $\boldsymbol{x}^+$ is the successor state and $f(\boldsymbol{x}, \boldsymbol{u})$ and $\boldsymbol{\epsilon} \in \mathbb{R}^{7\times1}$ are discrete-time forms of Eqs. (1), (4) and (5) with sampling time t_s. According to the related equations, $f(\boldsymbol{x}, \boldsymbol{u})$ and its partial derivatives are Lipschitz continuous. The disturbance $\boldsymbol{\epsilon}$ lies in the compact, convex set $\mathbb{E}$ that contains the origin. According to [26] the disturbances and uncertainties in space can be supposed as

$$|\boldsymbol{\epsilon}| \le \mu = c_1 + c_2|\boldsymbol{\omega}| + c_3|\boldsymbol{\omega}|^2; c_1, c_2, c_3 \ge 0 \tag{8}$$

This class of $\boldsymbol{\epsilon} \in \mathcal{K}$ which means it is strictly increasing, continuous and zeros at zero. The system is subject to the control torque constraint $\boldsymbol{u} \in \mathbb{U}$ where the compact set $\mathbb{U}$ is defined as

$$\boldsymbol{u} \in \mathbb{U}; \mathbb{U} \triangleq \left\{ \boldsymbol{u} \in \mathbb{R}^{3\times1} | \begin{matrix} -\mathrm{U} \le u_i \le \mathrm{U}; i = 1, 2 \\ u_i = 0; i = 3 \end{matrix} \right\} \tag{9}$$

where U is the magnitude of control input which can be generated by thrusters. It is supposed that $\boldsymbol{u}$ is the control input to the signal modulator if the on-off thruster is chosen as the actuator. According to the assumption, z axis is the unactuated axis. The associated nominal system of (7) is also defined as