Aerospace Science and Technology 41 (2015) 16–23

Contents lists available at ScienceDirect

Aerospace Science and Technology

www.elsevier.com/locate/aescte

Diving guidance via feedback linearization and sliding mode control

Luhua Liu [a], Jianwen Zhu [a], Guojian Tang [a], Weimin Bao [a,b]

[a] *College of Aerospace Science and Engineering, National University of Defense Technology, Changsha 410073, PR China*
[b] *China Aerospace Science and Technology Corporation, Beijing 100048, PR China*

ARTICLE INFO

Article history:
Received 2 April 2014
Received in revised form 5 October 2014
Accepted 20 November 2014
Available online 24 December 2014

Keywords:
Hypersonic vehicle
Dive phase
Precise guidance
Maneuver penetration
Feedback linearization
Sliding mode control

ABSTRACT

A novel guidance algorithm using feedback linearization and sliding mode control is proposed to realize high precision guidance and maneuvering flight for hypersonic vehicle in dive phase. First, the longitudinal ellipse trajectory which can satisfy both the terminal impact point and angle constraints is designed, and the lateral maneuver trajectory is also generated to improve the penetration capability. Second, it introduces feedback linearization to decouple the original motion equations into linear guidance subsystems in longitudinal and lateral channel. With the linear equations, the tracker is designed with the help of sliding mode control, and the practical tracking guidance law can also be obtained by substituting the tracker into the original system. In addition, the stability of the guidance system is proven using the *Lyapunov* theorem. This tracking guidance is independent of the relative motion information and relative equation, while only the current motion states are needed. Besides, the results of CAV-H vehicle guidance test show that maneuvering flight and high precision guidance can be realized with the novel proposed algorithm, and the robustness is also validated through different guidance missions and Monte Carlo simulation.

1. Introduction

With the rapid development of aerospace technology, hypersonic near-space vehicle flying at Mach 5 or above has became a hot research topic in the astronautic and aeronautic fields since the 1970s, such as the American common aero vehicle (CAV) [2,4,12,19]. There are several significant advantages for this new kind of vehicle, such as relatively high maneuverability, compared with the traditional ballistic missile, [19]. However, due to the high velocity, the dynamic model of hypersonic vehicle is fast time varying, highly nonlinear, and contains large parametric uncertainties [17]. Consequently, the guidance law which can satisfy terminal multiple constraints design remains a key technical challenge for a hypersonic vehicle, especially in dive phase.

Dive phase is the last one in the whole flight phases, and the guidance is concerned with steering the vehicle reach the designated termination target with prescribed condition while satisfying all the necessary path constraints for the operational considerations. Physically, this guidance objective can be achieved by controlling the direction of the aerodynamic lift force vector to satisfy the terminal multiple constraints [10]. There have been some considerable researches this problem, which can be divided into two classes. The first class based on zero line-of-sight (LOS) angle rate regards the guidance problem as the nonlinear control problem, which means that several advanced control theories such as feedback control, optimal control and sliding mode control can be well employed to make the flight states reach the required ones [6,8,15,16,18]. However, it will be unavailable when the movement information can't be measured in real time or the relative motion equations can't be constructed because the information and equations are essential to the above mentioned guidance method. The second one which is always called tracking guidance needs to design the ideal trajectories which can satisfy the terminal multiple constraints according to the current vehicle position and the target location, and proposes many different considerable algorithms, including feedback linearization to track the designed trajectories. Then the guidance objective can be well achieved once the trajectories can be tracked perfectly [7,9,11,13]. Furthermore, the two kinds of aforementioned guidance strategies can be combined together to realize high precision guidance because it is unnecessary to calculate relative information or construct the relative motion equations for the tracking guidance.

In addition, owing to the relatively decline of the flight altitude and velocity, the menace comes from ground air defense system is much more serious than glide phase. Furthermore, trajectory based on zero LOS angle rate is so straight that the penetrative ability will be greatly declined [14]. In order to enhance

---

E-mail address: zhujianwen1117@163.com (J. Zhu).

http://dx.doi.org/10.1016/j.ast.2014.11.014
1270-9638/

penetrability, it is necessary to design maneuver trajectories, such as weaving and barrel roll maneuver, and employ several different control methods to track the trajectories [1,3]. However, it's known that maneuvering flight can cause serious influence to the terminal guidance precision, and even lose the target [3]. Accordingly, maneuver and guidance commands should be combined together through weighted stacking to realize high precision guidance and maneuvering flight at the same time.

Motivated by the aforementioned problems, the current paper proposes a novel tracking guidance algorithm to achieve high precision guidance and maneuver penetration. It divides the three-dimensional diving movement into longitudinal and lateral directions and designs the corresponding guidance laws respectively. For longitudinal direction, the elliptic curve with excellent performance is adopted to design the trajectory which can satisfy the terminal impact point and angle constraints. Be different from the longitudinal motion, not only guidance mission but also maneuvering flight needs to the achieved in lateral direction. In order to reduce the tracking difficulty, it decouples the original nonlinear dynamic equations into the linear ones with the aid of feedback linearization, and employs sliding mode control (SMC) to track the generated trajectories with relatively high robustness and precision.

This paper is organized as follows. In Section 2, the simplified dynamic equations with reasonable assumptions are presented. In Section 3, the longitudinal elliptic trajectory and lateral maneuver trajectory are designed. Section 4 divided the nonlinear motion equations into linear ones via feedback linearization. In Section 5, based on the linear equations, SMC is introduced to the track to designed trajectories. In Section 6, simulation studies test the performance of the proposed guidance algorithm. Finally, conclusions are given in Section 7.

2. Formulation of motion

The three degree-of-freedom (3DOF) point-mass dynamics under ballistic coordinate system are adopted [10]. For the high lift-to-drag ratio vehicles with high flight velocity and short time in dive phase, the Carioles inertial force and centrifugal inertial force caused by the earth's self-rotation are very small, compared with the aerodynamic force and gravity, whose accelerations are approximately $10^{-2}g_0$ and $10^{-3}g_0$ respectively [5]. Therefore, the non-rotating spherical earth assumption is employed in the guidance law design. The motion equations can then be simplified as follows:

$$\begin{cases} \dot{v} = -\dfrac{\rho v^2 S_m C_D}{2m} - g\sin\theta \\ \dot{\theta} = \dfrac{\rho v^2 S_m C_L \cos\upsilon}{2mv} - \dfrac{g\cos\theta}{v} + \dfrac{v\cos\theta}{r} \\ \dot{\sigma} = -\dfrac{\rho v^2 S_m C_L \sin\upsilon}{2mv\cos\theta} + \dfrac{v\tan\phi\cos\theta\sin\sigma}{r} \\ \dot{\phi} = \dfrac{v\cos\theta\cos\sigma}{r} \\ \dot{\lambda} = -\dfrac{v\cos\theta\sin\sigma}{r\cos\phi} \\ \dot{r} = v\sin\theta \end{cases} \tag{1}$$

Here the position coordinates are the radial distance from the center of the Earth to the vehicle r, the longitude λ, the latitude φ; and the velocity coordinates are the earth-relative velocity magnitude v, the velocity slope angle θ, and velocity azimuth angle σ measured from the north in a clockwise direction. ρ is atmospheric density; m is vehicle mass; S_m is reference area; $g = \mu_M/r^2$ is the gravity, μ_M is the gravitational constant of the earth. C_D and C_L are aerodynamic drag and lift coefficients, respectively. Among these coefficients, there is one control variable angle of attack α, whereas the other control variable is the bank angle υ.

For simplification, Eq. (1) can be further divided into longitudinal and lateral directions. The longitudinal equations are:

$$\begin{cases} \dot{v} = -\dfrac{\rho v^2 S_m C_D}{2m} - g\sin\theta \\ \dot{\theta} = \dfrac{\rho v^2 S_m C_L \cos\upsilon}{2mv} - \dfrac{g\cos\theta}{v} + \dfrac{v\cos\theta}{r} \\ \dot{h} = v\sin\theta \\ \dot{L} = R_e v\cos\theta/(R_e + h) \end{cases} \tag{2}$$

Here $h = r - R_e$ is the flight altitude; L is the range. For the hypersonic vehicles flight in near space, there exists $h \ll R_e$, then the range derivative can be further simplified as:

$$\dot{L} = v\cos\theta \tag{3}$$

The lateral equation is:

$$\dot{\sigma} = -\frac{\rho v^2 S_m C_L \sin\upsilon}{2mv\cos\theta} + \frac{v\tan\phi\cos\theta\sin\sigma}{r} \tag{4}$$

with the simplified equations shown in Eqs. (2) and (4), the reference trajectories and the corresponding tracker can be designed, which means the guidance goal will be well achieved.

3. Diving trajectories design

The generation of the diving trajectory can be described as: given the simplified motion model shown in Eq. (2), maneuver parameters and terminal required states, obtain the feasible trajectories in longitudinal and lateral directions, respectively, which can steer the vehicle to attack the ground fixed target and realize maneuvering flight along with the path constraints.

3.1. Longitudinal trajectory

It is known that high precision guidance is the chief task for hypersonic vehicle in dive phase. To begin with, we should design the longitudinal trajectory which can satisfy the terminal impact point constraint:

$$\lim_{h\to 0} L(h) = L_f \tag{5}$$

Here h is the flight altitude, and L_f is the terminal range in dive phase. Furthermore, the terminal impact point and angle constraints can be expressed as:

$$\begin{cases} \lim\limits_{L\to L_f} h(L) = 0 \\ \lim\limits_{L\to L_f} \theta(L) = \theta_f \end{cases} \tag{6}$$

It is known that there are three parameters for the ellipse with longitudinal excursion: longitudinal offset, major and minor semi axis. In addition, the ellipse curve is so smooth that make it is much easier to be tracked. Therefore, with the three terminal constraints: range, altitude and impact angle, the entire ellipse parameters can be calculated easily, which means the longitudinal elliptic trajectory can be determined.

To begin with, as shown in Fig. 1, longitudinal planar coordinate system with the origin O locating on the crossover point of the earth's surface and ligature between center of the Earth and vehicle's initial position, the positive x-axis pointing to the target T, and the positive y-axis pointing upward is constructed to describe