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ORIGINAL ARTICLE

Fractional single-phase-lagging heat conduction model for describing anomalous diffusion

T.N. Mishra[a,*], K.N. Rai[b]

[a]DST-CIMS, Faculty of Sciences, BHU, Varanasi, UP 221005, India
[b]Mathematical Sciences, IIT BHU, Varanasi, UP 221005, India

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Anomalous diffusion

Abstract The fractional single-phase-lagging (FSPL) heat conduction model is obtained by combining scalar time fractional conservation equation to the single-phase-lagging (SPL) heat conduction model. Based on the FSPL heat conduction model, anomalous diffusion within a finite thin film is investigated. The effect of different parameters on solution has been observed and studied the asymptotic behavior of the FSPL model. The analytical solution is obtained using Laplace transform method. The whole analysis is presented in dimensionless form. Numerical examples of particular interest have been studied and discussed in details.

1. Introduction

The Fourier law of heat conduction assumes that heat flux vector $\boldsymbol{q}(\boldsymbol{r}, t^*)$ and temperature gradient $\nabla T(\boldsymbol{r}, t^*)$ appear at the same time instant t^* and consequently implies that thermal signal propagates with an infinite speed. The infinite speed of heat propagation, implying that a thermal disturbance applied at a certain location in a medium, can be sensed immediately anywhere else in the medium [1], it is one of the drawbacks of Fourier's law. Many models [2–4] do not agree with the Fourier law since this law is based on infinite speed of heat propagation and simultaneous development of heat flux and temperature gradient.

Cattaneo [5] and Vernotte [6] removed the deficiency of Fourier law by adding a lagging parameter in Fourier law

*Corresponding author. Tel.: +91 9198006395.
E-mail address: t.mishra01@gmail.com (T.N. Mishra).

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Nomenclature

c	thermal wave propagation speed (unit: m/s)
c_b	specific heat capacity (unit: J/(kg · K))
g^*	internal heat generation (unit: W/m^3)
g	dimensionless internal heat generation ($g=4\alpha_1 g^*/cI_r$)
h	real or complex valued function
I_r	reference heat flux
k	thermal conductivity (unit: W/(m · K))
L	Laplace transform
L^{-1}	inverse Laplace transform
P_d	Predvoditelev number ($P_d=b_1 l^2/\alpha_1 \Delta T$)
q^*	dimensionless heat flux ($q^*=q/I_r$)
q	heat flux (unit: W/m^2)
$\boldsymbol{r}$	position vector
t^*	time (unit: s)
t	dimensionless time ($t=t^*(c^2/2\alpha_1)^{\alpha-1}$)
$t_p{}^*$	impulse time (unit: s)
t_p	dimensionless impulse time ($t_p=t_p{}^*(c^2/2\alpha_1)^{\alpha-1}$)
T	temperature (unit: K)
∇T	temperature gradient (unit: K/m)
x	dimensionless spatial coordinate ($x=cy/2\alpha_1$)
y	spatial coordinate (unit: m)

Greek letters

α_1	thermal diffusivity (unit: m^2/s)
α	order of fractional derivative lies in (0, 1]
θ	dimensionless temperature ($\theta=kcT/\alpha_1 I_r$)
ρ	density (unit: kg/m^3)
τ^*	lagging parameter (unit: s)
τ	dimensionless lagging parameter ($\tau=\tau^*(c^2/2\alpha_1)^{\alpha-1}$)

and proposed the CV constitutive relation in the form of

$$\tau^* \frac{\partial \boldsymbol{q}}{\partial t} + \boldsymbol{q} = -k\nabla T \tag{1}$$

where k is the thermal conductivity of the medium and τ is the material property called the lagging time. This model characterizes the combined diffusion and wave like behavior of heat conduction and predicts a finite speed

$$c = \left(\frac{k}{\rho c_b \tau^*}\right)^{\frac{1}{2}} \tag{2}$$

for heat propagation [7], where ρ is the density and c_b is the specific heat capacity. For more details about thermal lagging in wave theory see [8,9]. Tzou [10–14] generalized the CV heat conduction model [1] as

$$\boldsymbol{q}(\boldsymbol{r}, t^* + \tau^*) = -k\nabla T(\boldsymbol{r}, t^*) \tag{3}$$

The above Eq. (3) is known as the single-phase-lagging (SPL) heat conduction model. This model establishes the heat flux (the result) when a temperature gradient (the cause) is suddenly imposed.

In the past decades, the phenomena of anomalous diffusion have been observed in numerous physical and biological systems [15–22]. The anomalous diffusion is characterized by diffusion constant and the mean square displacement of diffusing species in the form

$$\langle x^2(t)\rangle = t^\alpha, \quad t\to\infty$$

where α is the diffusion exponent. This phenomena is usually divided into anomalous subdiffusion for $0<\alpha<1$ and anomalous superdiffusion for $1<\alpha<2$. If $\alpha=1$, we have the diffusion. To investigate diffusion phenomenon there are numerous approaches have been used [23–26]. Antonakakis et al. [27] analytically solved the diffusion equation with Gaussian heat source. Nishikawa [28] constructed the first, second and third order finite volume scheme for diffusion equation. This method enables straightforward constructions of diffusion schemes for finite-volume methods on unstructured grids.

Now concerning fractional heat transfer, the topic is to some extent new. Povstenko [29] proposed fractional heat equation for modeling thermoelasticity and Ezzat [30] investigated heat transfer in MHD for a thermoelectric medium. Compte and Metzler [31] proposed three possible time fractional generalizations of Cattaneo model, which are supported by continuous-time random walks, non-local transport theory and delayed flux-force relations, respectively. Povstenko [32] studied the time fractional Cattaneo type equations and formulated the corresponding theories of thermal stresses. Qi and Jiang [33] and Atanackovic et al. [34] built the Cattaneo-type space-time fractional heat conduction equation, and the explicit solutions of the Cauchy problem are given in terms of a series and integral representation. Ghazizadesh et al. [35] solved the fractional Cattaneo equation by explicit and implicit finite difference schemes. Qi et al. [36] are used the generalized Cattaneo model with fractional derivative and solved the corresponding Cattaneo-type fractional heat conduction equation for laser heating by Laplace transforms technique. Very recently, Ghazizadesh et al. [37] generalized the SPL heat conduction model to the FSPL heat conduction model. They obtained the FSPL heat conduction model by applying fractional Taylor series formula [38] on the SPL delayed equation.

In the present study, the fractional single-phase-lagging (FSPL) heat conduction model is obtained by combining the SPL model to the fractional conservation equation and studied the combined effects of anomalous subdiffusion and anomalous superdiffusion. The effects of fractional order (α) and heat source (g) on temperature distributions within the thin film based on the FSPL heat conduction model will also be investigated and studied the asymptotic behavior (i.e. $\alpha\to 1$) of the FSPL model. The outline of the paper is as follows. In Section 2, FSPL heat conduction model is presented. Solution is given in Section 3. Section 4 contains results and discussion. Concluding remarks are summarized in Section 5.