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Non-Gaussian properties of second-order wave orbital velocity

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ABSTRACT

A stochastic second-order wave model is applied to assess the statistical properties of wave orbital velocity in random sea states below the water surface. Directional spreading effects as well as the dependency of the water depth are investigated by means of a Monte-Carlo approach. Unlike for the surface elevation, subharmonics dominate the second-order contribution to orbital velocity. We show that a notable set-down occurs for the most energetic and steepest groups. This engenders a negative skewness in the temporal evolution of the orbital velocity. A substantial deviation of the upper and lower tails of the probability density function from the Gaussian distribution is noticed; velocities are faster below the wave trough and slower below the wave crest when compared with linear theory predictions. Second-order nonlinearity effects strengthen with reducing the water depth, while weaken with the broadening of the wave spectrum. The results are confirmed by laboratory data. Corresponding experiments have been conducted in a large wave basin taking into account the directionality of the wave field. As shown, laboratory data are in very good agreement with the numerical prediction.

1. Introduction

Accurate wave statistics is crucial to establish concise predictions as well as realistic design values for wave heights and wave-induced velocities. They can provide a good estimation on the air gap of fixed and tension leg offshore platforms. Velocities, in particular, are the primary input for wave-induced loads on surface and subsurface structures Morison et al. (1950); Dean and Perlin (1986); Faltinsen (1993); Dean and Dalrymple (2000). Nearshore, in a regime of finite water depth, wave kinematics significantly affect sediment transport processes Crawford and Hay (2001); Greenwood (2003); Myrhaug et al. (2015).

Provided that waves are of small amplitude, i.e. assuming gentle sloping, the nonlinear water wave problem can be linearised and the irregular sea surface may then be reconstructed by a linear superposition of sinusoidal components Dean and Dalrymple (2000). In statistical terms, this implies that waves can be considered as a stationary, ergodic and Gaussian random process. Assuming the process to be narrow-banded, it is known that wave amplitudes satisfy the Rayleigh distribution. However, wave steepness is often too large for the linear theory to be valid in studying ocean waves in deep and coastal waters in a general framework. In terms of current design practice, second-order nonlinear contributions are applied to account for the mutual interaction between wave components Forristall (2000). General second-order corrections to linear solutions for the surface elevation η and velocity potential ϕ are given in Sharma and Dean (1981). Note that second-order quasi-deterministic solutions (see for instance Boccotti (2000)) may be more appropriate for particularly high amplitude waves.

With respect to the water surface elevation, second-order nonlinearity generates high-frequency bound modes (super-harmonics), which make wave crests higher and sharper while troughs are flatter and less deep compared to linear models. It also induces low-frequency components (sub-harmonics), which produce a set-down under the most energetic wave groups Forristall (2000); Toffoli et al. (2007). Taking into account the fact that super-harmonics induce a dominant contribution Toffoli et al. (2007), the probability density function (*p.d.f.*) of the surface elevation is then characterised by a positive skewness and substantial deviations of the upper and lower tails from the Normal (Gaussian) distribution Forristall (2000); Tayfun (1980); Tayfun and Fedele (2007). Deviations from Normality are reduced by directional spreading in deep-water and enhanced in finite water depths Forristall (2000).

For the velocity potential, and consequently wave orbital velocities, second-order super- and sub-harmonics are of the same order of magnitude nearby the mean water level. Nevertheless, below

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the surface, super-harmonics decay rapidly Dean and Perlin (1986), while sub-harmonics retain a significant fraction of their energy Dean and Perlin (1986); Baldock and Swan (1996); Ning et al. (2009); Johannessen (2010). Consequently, sub-harmonics produce a general increase of velocity below the troughs and decrease it below the crests for the most energetic wave groups. This effect amplifies with the distance from the surface Romolo et al. (2014). Song and Wu (2000) noted, numerically, that the *p.d.f.* of orbital velocity becomes negatively skewed with respect to the depth. This result, however, is neither confirmed by laboratory experiments nor by field observations Battjes and Van Heteren (1980); Drennan et al. (1992); Sultan and Hughes (1993); You (2009).

The effect of second-order nonlinear contribution on wave orbital velocity still remains unclear. As an example, it is not straightforward yet whether second-order variations to velocities are sufficiently strong to induce deviations of the upper and lower tails of the *p.d.f.* from the Gaussian distribution. Furthermore, the effect of wave directionality (wave directional spreading) has not been properly assessed yet.

The paper is structured as follows. First, we revisit the contribution of second-order nonlinearity on wave orbital velocities with a stochastic second-order model Sharma and Dean (1981). A brief analytical discussion of the second-order interaction kernels and effects on regular waves (both mono- and bi-chromatics) is discussed in the next section. In the following Section 3 the stochastic model and its initial conditions are presented. Results of Monte-Carlo simulations for unidirectional and directional wave fields are assessed in order to evaluate departures from the Gaussian distribution, with particular focus on extreme values, i.e. deviations of the lower and upper tail of the distribution. A comparison with experimental velocity field data, collected in a large directional basin in infinite and finite depth conditions Toffoli et al. (2013), is also discussed. Final remarks and a discussion with respect to the main reported results are presented in the Conclusions.

2. Second-order wave orbital velocity

2.1. Interaction kernels

Taking into account the second-order of nonlinearity, the velocity potential can be written as a sum of the linear solution of the Euler equations for surface gravity water waves ($\phi^{(1)}$) and a second-order correction consisting of super- and sub-harmonics, denoted by $\phi^{(2+)}$ and $\phi^{(2-)}$, respectively Sharma and Dean (1981). Under the hypothesis of inviscid fluid and irrotational potential flow, the linear velocity potential of a finite number of M modes, which correspond to number of elements used in the numerical discretisation, in a water of arbitrary depth is

$$\phi^{(1)} = \sum_{i=1}^{M} \frac{a_i g}{\omega_i} \frac{\cosh[k_i(d+z)]}{\cosh[k_i d]} \sin\Theta_i \quad (1)$$

where g is the gravitational acceleration, d the water depth, z the vertical coordinate (origin $z = 0$ at the mean water level and positive upwards), a_i the amplitude of the i−th wave component, ω_i the concurrent angular frequency and $k_i = |\mathbf{k}_i|$ the concurrent wavenumber. $\Theta_i = \mathbf{k}_i \cdot \mathbf{x} - \omega_i t + \varepsilon_i$ where ε_i denotes the arbitrary phase.

The second-order correction is described as Sharma and Dean (1981)

$$\phi^{(2\pm)} = \frac{1}{4}\sum_{i=1}^{M}\sum_{j=1}^{M} \frac{a_i a_j g^2}{\omega_i\omega_j} \frac{\cosh[k_{ij}^{\pm}(d+z)]}{\cosh[k_{ij}^{\pm} d]} K_{\phi}^{\pm} \sin(\Theta_i \pm \Theta_j), \quad (2)$$

where $K_\phi^\pm$ are the positive and negative kernels:

$$K_\phi^\pm = \frac{D_{ij}^\pm}{\omega_i \pm \omega_j} \quad (3)$$

with

$$D_{ij}^{+} = \frac{\left(\sqrt{R_i}+\sqrt{R_j}\right)^2\left[\sqrt{R_i}\left(k_j^2-R_j^2\right)+\sqrt{R_j}\left(k_i^2-R_i^2\right)\right]}{\left(\sqrt{R_i}+\sqrt{R_j}\right)^2-k_{ij}^{+}\tanh\left(k_{ij}^{+}d\right)} + \frac{2\left(\sqrt{R_i}+\sqrt{R_j}\right)^2\left(k_i\cdot k_j-R_iR_j\right)}{\left(\sqrt{R_i}+\sqrt{R_j}\right)^2-k_{ij}^{+}\tanh\left(k_{ij}^{+}d\right)} \quad (4)$$

$$D_{ij}^{-} = \frac{\left(\sqrt{R_i}-\sqrt{R_j}\right)^2\left[\sqrt{R_j}\left(k_i^2-R_i^2\right)-\sqrt{R_i}\left(k_j^2-R_j^2\right)\right]}{\left(\sqrt{R_i}-\sqrt{R_j}\right)^2-k_{ij}^{-}\tanh\left(k_{ij}^{-}d\right)} + \frac{2\left(\sqrt{R_i}-\sqrt{R_j}\right)^2\left(k_i\cdot k_j+R_jR_j\right)}{\left(\sqrt{R_i}-\sqrt{R_j}\right)^2-k_{ij}^{-}\tanh\left(k_{ij}^{-}d\right)} \quad (5)$$

$R_i = k_i tanh(k_i d)$, $k_{ij}^\pm = |\mathbf{k}_i \pm \mathbf{k}_j|$ and $\mathbf{k} = (k_x, k_y)$.

The sums in Eq. (2) are computed directly, no de-aliasing is required. Wave orbital velocities are then calculated as the partial derivative of the velocity potential:

$$u = \frac{\partial\phi}{\partial x};\ v = \frac{\partial\phi}{\partial y};\ \text{and}\ w = \frac{\partial\phi}{\partial z} \quad (6)$$

with u and v the horizontal components and w the vertical component.

The kernel functions for two interacting components with wave numbers k_1 and k_2 are presented in Fig. 1 for deep ($k_1 d \to \infty$) and finite ($k_1 d = 1.29$) water depth conditions. A key feature for the limiting case $k_1 d \to \infty$ is a non-existent contribution of the positive kernel for collinear modes, as shown in Fig. 1 (a), meaning that super-harmonics are not generated by mutual wave-wave interaction Dean and Dalrymple (2000); Kim (2008). A slight, but yet negligible, contribution of the positive kernel still occurs when the components are non-collinear (Fig. 1 (b)).

In Fig. 1 (c), we notice on the other hand that the negative kernel significantly contributes to the second-order nonlinearity, when interacting components are collinear; this for any relative water depth. The interaction is particularly strong for components of similar frequency, while it decays rapidly for increasing frequency difference. There is no dependence of the negative kernel on directionality, as observed in Fig. 1 (d). However, it is important to note that the negative kernel tends to infinity for self-interactions. Numerically, this self-interaction is resolved by forcing the kernel to be equal to zero Dean and Dalrymple (2000); Kim (2008). As expected, sub-harmonics strengthen as the relative water depth is reduced.

2.2. Second-order orbital velocity for regular waves

A preliminary assessment of the effect of the interaction kernel is presented here for regular waves: (*i*) a self-interacting monochromatic wave; (*ii*) two interactive collinear monochromatic waves of different frequency. As noted in Song and Wu (2000), second-order components u, v and w are statistically dependent. For simplicity, only the horizontal component u along the main propagation direction will be considered hereafter. Numerically, the u component is derived from the velocity potential in Fourier space as $\mathcal{F}^{-1}[ik_x\mathcal{F}(\phi)]$,