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Theoretical analysis and numerical simulation of mechanical energy loss and wall resistance of steady open channel flow*

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Abstract: The mechanical energy loss and the wall resistance are very important in practical engineering. These problems are investigated through theoretical analysis and numerical simulation in this paper. The results are as follows. (1) A new mechanical energy equation for the total flow is obtained, and a general formula for the calculation of the mechanical energy loss is proposed. (2) The general relationship between the wall resistance and the mechanical energy loss for the steady channel flow is obtained, the simplified form of which for the steady uniform channel flow is in consistent with the formula used in Hydraulics deduced by π theorem and dimensional analysis. (3) The steady channel flow over a backward facing step with a small expansion ratio is numerically simulated, and the mechanical energy loss, the wall resistance as well as the relationship between the wall resistance and the mechanical energy loss are calculated and analyzed.

Key words: channel flow, energy equation, mechanical energy loss, resistance

Introduction

Channel flows are gravity driven flows, with viscous resistance and form resistance at the channel wall to induce mechanical energy loss when the liquid flows downstream. The mechanical energy loss and the wall resistance are important issues in Hydraulics and Engineering Fluid Mechanics, as well as in practical engineering. The wall resistance was studied by experiments[1,2], numerical simulations[3,4] and theoretical analyses[5,6] since the pioneering work of the famous German scholar, Prandtl.

Among the experimental studies, Knight and Sterling[1] conducted experiments to determine the distribution of the boundary shear stress, which, they found, was related to the shape of the secondary flow cells while the shape of the secondary flow cells was decided by the aspect ratio. Yoon et al.[7] studied the velocity distribution and the resistance coefficient with circular flume experiments. It is shown that the flow depth significantly affects the velocity distribution. The ratio of mean to maximum velocities will reduce and the position of the maximum velocity will be lowered if the flow depth is below 50%. The wall friction and the Manning coefficients also differ from the generally estimated values as the flow depth is reduced by 50%. Patnaik et al.[8] conducted experiments in highly sinuous trapezoidal meandering channels to investigate the effect of the aspect ratio and the sinuosity on the wall resistance under the smooth and rigid bed condition. The percentage of the shear force on the inner wall, the outer wall and the bed were estimated and the experimental data were used to establish an equation for the percentage of the total wall shear force, which is more consistent and covers a wider range of aspect ratio than available ones. The wall resistance was also studied by numerical simulations. Cacqueray et al.[3] investigated the shear stress in a smooth rectangular channel by numerical simulations and it is shown that the stress associated with the secondary flow and the shear stress on the interface could not be neglected and the provided division lines match well with the existing results. Berlamont et al.[9] studied the shear stress distribution in circular channels by numerical simulations, focusing on the effect of the aspect ratio, the velocity distribution and the wall roughness on the stress distribution. The computational fluid dynamics (CFD) was used to determine the

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distribution of the bed and sidewall shear stresses in trapezoidal channels by Ansari et al.[10] and the effects of the slant angle of the side walls, the aspect ratio and the composite roughness on the shear stress distribution were analyzed. The stress associated with the secondary flow and the shear stress on the interface are the main contributions. The distribution of the shear stress on the boundaries is considerably influenced by the variation of the slant angle and the aspect ratio, especially for low aspect-ratio channels. Stoesser et al.[11] calculated the turbulent flow in a meandering channel with the steady Reynolds-averaged Navier-Stokes equations (RANS) code based on an isotropic turbulence closure and the large eddy simulation (LES) code, respectively and the results were compared with the previous ones. It is shown that the RANS code ove-predicts the size and the strength of the secondary cell while the LES code is better in the secondary cell prediction. However, the wall shear stresses obtained by the LES code and the RANS code agree well though the secondary cells have a great influence on the distribution of the wall shear stress. In addition, the wall resistance was studied by analytical methods. For example, Guo and Julien[12] discussed the wall shear stress and Yang and Lim[13] discussed this paper, subsequently. Khodashenas et al.[14] made a comparison among six methods for the determination of the boundary shear stress distribution. There were a few investigations of the wall mechanical energy loss. Liu et al.[15,16] established a mechanical energy equation for the total flow of incompressible homogeneous liquid in pipes and open channels, and the expressions for the mechanical energy loss were suggested based on a theoretical analysis. In Ref.[16] the effect of the aspect ratio and the Reynolds number on the mechanical energy loss in open channels was studied numerically, and it is shown that in a laminar flow, the coefficient of the mechanical energy loss decreases with the increase of the aspect ratio and the Reynolds number, yet in a turbulent flow, this coefficient tends to be independent of the aspect ratios as long as the Reynolds number is large enough. Nevertheless, the mechanical energy defined in Ref.[16] includes the mean turbulent kinetic energy, which is difficult to determine in practical engineering now and the relationship between the wall resistance and the mechanical energy loss is not available.

Thence in this paper the investigations are focused on the following points: (1) A new mechanical energy equation for the total flow is proposed by defining the mechanical energy as the sum of the potential energy of gravity, the potential energy of surface force and the mean kinetic energy. (2) The general relationship between the wall resistance and the mechanical energy loss in open channel flows is analyzed, and further discussed for the steady uniform flow and the steady flow over a backward-facing step.

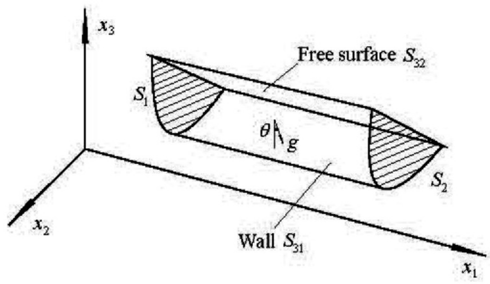

Fig.1 The sketch of open channel flow

1. Improved mechanical energy equation for steady channel flow

We consider the steady channel flow of a control volume V as shown in Fig.1, with the outer boundaries S composed of two cross-sections S_1 and S_2 at its upstream and downstream positions with a distance L between them, the channel wall S_{31} and the free surface S_{32}. For the statistically steady turbulent flow of homogeneous incompressible liquid of density ρ in the gravitational field, the mean surface force $\overline{T_{ij}} = -\overline{p}\delta_{ij} + \overline{\tau_{ij}}$, where $\overline{p}$ and $\overline{\tau_{ij}}$ are the mean pressure and the mean viscous shear stress respectively can be expressed by using the Reynolds averaged equation (Eq.(7) of this paper) as

$$\frac{\partial(\overline{T_{ij}u_i})}{\partial x_j} = \overline{T_{ij}s_{ij}} + \rho\overline{u_i}\left(\frac{\partial \overline{u_i u_j}}{\partial x_j} - f_i + \frac{\partial \overline{u_i' u_j'}}{\partial x_j}\right) \tag{1}$$

where $\overline{u_i}$, $\overline{s_{ij}}$ and f_i are the mean velocity, the rate of the mean deformation and the unit mass force, respectively. We have

$$-\rho\overline{u_i}f_i = \frac{\partial}{\partial x_i}[\rho\overline{u_i}(gx_3\cos\theta - gx_1\sin\theta)] \tag{2a}$$

$$\overline{T_{ij}s_{ij}} = \overline{\tau_{ij}s_{ij}} \tag{2b}$$

$$\rho\overline{u_i}\frac{\partial\overline{u_i u_j}}{\partial x_j} = \frac{\partial}{\partial x_i}\left(\frac{1}{2}\rho\overline{u_i u_j u_j}\right) \tag{2c}$$

Therefore, Eq.(1) can be simplified further as

$$\frac{\partial}{\partial x_i}\left[\rho\overline{u_i}\left(gx_3\cos\theta - gx_1\sin\theta + \frac{p_s}{\rho} + \frac{1}{2}\overline{u_j u_j}\right)\right] =$$