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Scattering of gravity waves by a porous rectangular barrier on a seabed*

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Abstract: Within the frame of linear potential flow theory, the impact of a porous rectangular barrier on a seabed on the dynamic characteristics of gravity waves is investigated. The porous barrier can be regarded as an abstract representative such as a seabed plant, a wave breaker, inhomogeneous topography, and trussed supporting of ocean engineering platform, etc. In the process of mathematical modeling, the method of matched eigenfunction expansions is employed for analysis, where a newly defined form of inner product is introduced to improve the simplicity of derivation. Under this definition, the inner product is automatically orthogonal, which will provide great simplification to obtain the expansion coefficients. Once the wave numbers for the fluid region and the barrier region are obtained, the reflection and transmission coefficients of the wave motion can readily be calculated.

Key words: gravity waves, dispersion relation, permeability, orthogonality

Introduction

Considering the impact of a porous medium on wave motions is one of the meaningful and significant research issues in hydrodynamics. Many types of constructs and topographies in the ocean environment, such as wave breaker, permeable seabed, water plants zone, etc., can be modeled by porous media. The permeability of relevant objects is usually formulated in two different ways. One is to neglect the thickness of the porous medium by considering the changes of velocity field on its surface to carry out appropriate boundary conditions, e.g., Martha et al.[1] and Mohapatra[2]. Another way is to regard the porous medium as a special region where the wave motion is described by a potential function after equivalent averaging. For example, Das and Bora[3] studied the case of wave propagating across a porous medium occupying the whole depth of the fluid and afterwards reflected by a vertical rigid wall. Recently, Metallinos et al.[4] investigated wave scattering by a porous trapezoidal structure submerged on the seabed of shallow water region by a computational method.

In this Letter, an improved semi-analytical approach is proposed to obtain the reflection and transmission coefficients of free-surface gravity wave scattering by a porous rectangular barrier fixed on the fluid bottom. The method of matched eigenfunction expansions is employed in the derivation. A newly defined form of inner products with orthogonality for the vertical eigenfunctions in the porous barrier region is found to deal with the matching relations between different fluid regions, by which the number of simultaneous equations is reduced by half.

The physical model of the related problem is given under a two-dimensional Cartesian coordinate system xoz placed on the surface of a homogeneous fluid with the z axis upwards vertically, as shown in Fig.1. Free-surface gravity waves income from $-\infty$ onto the porous rectangular barrier symmetrically mounted on the seabed with respect to the z axis. The width and the height of the barrier are $2b$ and h_2,

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respectively, and the distance from the top of the barrier to the free surface is h_1. Let H be the depth of the fluid. Obviously $H = h_1 + h_2$.

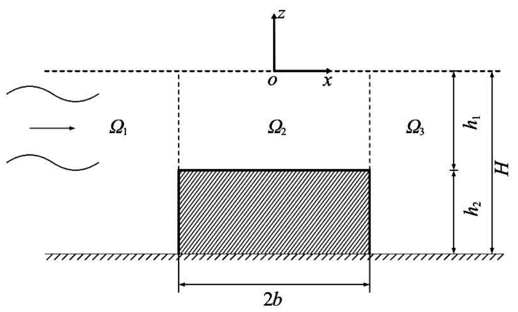

Fig.1 Schematic diagram of free surface gravity waves incoming onto a porous rectangular barrier mounted on seabed

With the assumptions that the fluid is ideal and incompressible and the wave motion is irrotational and time-harmonic, the velocity field of the fluid can be described within the potential flow theory. Let $\Phi(x, z, t)$ be the velocity potential function, where t is the time. The fluid inside the porous barrier still satisfies the conservation law of mass, such that the wave motion inside can be described by the potential flow theory as well. By regarding $\Phi(x,z,t)$ as a piecewise function for the whole fluid domain, the part inside the barrier is actually generated by the equivalent averaging of the relevant fluid motion. Let ω be the frequency of the incident waves. We separate the time variable of $\Phi(x,z,t)$ via taking the form of $\mathrm{Re}\{\phi(x,z)\mathrm{e}^{-\mathrm{i}\omega t}\}$, such that $\phi(x,z)$, the spatial part of the potential function, should satisfy

$$\nabla^2\phi(x,z) = 0 \tag{1}$$

On the impermeable seabed, the boundary condition reads

$$\frac{\partial\phi}{\partial z} = 0 \quad (z = -H) \tag{2}$$

According to the linearized Bernoulli equation, the combined boundary condition on the free surface is

$$\frac{\partial\phi}{\partial z} - \frac{\omega^2}{g}\phi = 0 \quad (z = 0) \tag{3}$$

where g is the gravitational acceleration.

Let B denote the boundary line $(x = \pm b, -H < z < -h_1) \cup (-b < x < b,\ z = -h_1)$. The velocity and pressure must be coincident beside B, respectively. Sollitt and Cross[5] introduced three dimensionless parameters to describe the physical characteristics of the porous medium, i.e., the porosity ε, the linear friction factor f and the inertial term s, by which the continuity conditions on B are given as follows:

$$\frac{\partial\phi_f}{\partial n} = \varepsilon\frac{\partial\phi_p}{\partial n} \quad \text{on} \quad B \tag{4}$$

$$\phi_f = G\phi_p \quad \text{on} \quad B \tag{5}$$

where the subscripts "f" and "p" are applied to distinguish the potentials outside and inside the boundary line, respectively, $\partial/\partial n$ refers to the directional derivative along the outward normal vector of B. According to the form of the general solution we employed, G is calculated from $s + \mathrm{i}f$. On the boundaries ($x = \pm b$, $-h_1 < z < 0$), the continuities of velocity and pressure also give

$$\frac{\partial\phi(\pm b^-, z)}{\partial x} = \frac{\partial\phi(\pm b^+, z)}{\partial x} \tag{6}$$

$$\phi(\pm b^-, z) = \phi(\pm b^-, z) \tag{7}$$

Associating Eqs.(4)-(7), the matching relations can be derived as

$$\frac{\partial\phi(\pm b^\pm, z)}{\partial x} = \frac{\partial\phi(\pm b^\mp, z)}{\partial x} \quad (-h_1 < z < 0) \tag{8a}$$

$$\frac{\partial\phi(\pm b^\pm, z)}{\partial x} = \varepsilon\frac{\partial\phi(\pm b^\mp, z)}{\partial x} \quad (-H < z < -h_1) \tag{8b}$$

$$\phi(\pm b^\pm, z) = \phi(\pm b^\mp, z) \quad (-h_1 < z < 0) \tag{9a}$$

$$\phi(\pm b^\pm, z) = G\phi(\pm b^\mp, z) \quad (-H < z < -h_1) \tag{9b}$$

Let Ω_1, Ω_2 and Ω_3 represent the regions $x < -b$, $-b < x < b$ and $x > b$, respectively, as shown in Fig.1. Substituting the general solution of Laplace's equation into Eqs.(2) and (3), the vertical eigenfunction $Z(k,z)$ and the dispersion relation for the regions Ω_1 and Ω_3 can be obtained as follows:

$$Z(k,z) = \frac{\cosh k(z+H)}{\cosh kH} \tag{10}$$

$$\omega^2 = gk\tanh kH \tag{11}$$