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ScienceDirect
Journal of Hydrodynamics
2016,28(1):166-169
DOI: 10.1016/S1001-6058(16)60618-2

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science/journal/l0016058

Ski-jump trajectory based on take-off velocity*

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(Received December 10, 2015, Revised January 18, 2016)

Abstract: The theoretical method estimating ski-jump trajectory was paid attention to and modified. The present method is based on the effects of the take-off velocity and the angle in the sensitivity analysis of parameters. The experiments are conducted for a triangular-shaped flip bucket in order to reveal the relationships between the take-off velocity and its influencing factors. The results show that, the take-off velocity has a much larger effect on the impact point than the take-off angle. The take-off velocities of both upper and lower trajectories are all functions of the approach flow Froude number, the deflector height and the deflection angle, especially, the results of the deflection angle of 25° could be directly used when this angle is larger than 25°. Meanwhile, this method is checked and the maximum relative errors of both $x_{\mathrm{U\,cal}}$ and $x_{\mathrm{L\,cal}}$ are 5.1% and 5.6%, respectively.

Key words: trajectory, triangular-shaped flip bucket, take-off velocity, sensitivity analysis of parameters

In the designs for the ski-jump type energy dissipation, the theory of the projectile for a rigid body is often used to estimate the jet trajectory[1]. This theory involves two basic hypotheses in the estimation of the trajectory. One is that the water is a rigid body, and the other is that the air resistance can be neglected. Thus, in certain engineering projects, a deviation about 20%-60% is often found if the theory of the projectile for a rigid body is directly applied[2].

The sources of errors of the theoretical method include the effects of the air resistance, the difference between the take-off angle of the upper or lower trajectory and the deflection angle, and the changes of the take-off velocity of the upper or lower trajectory with the approach flow velocity. Much effort was made to improve the methodology of estimating the trajectory.

For the effect of the air resistance, Liu et al., based on a theoretical analysis, proposed an air resistance coefficient for modifying the estimation of the trajectory, but this coefficient involves some factors, such as the densities of both the air and jet flows, the resistance, and an unknown function of the take-off angle, so this cannot be conveniently used due to some unknown variables[3]. Further, for circular-shaped flip buckets, Wu et al. proposed expressions of the air resistance coefficient of the upper and lower trajectories, which could be used directly in the estimations of the trajectory[4].

With respect to the difference between the take-off and deflection angles, Steiner et al.[5] and Heller et al.[6] observed that the take-off angle of upper or lower trajectory is clearly smaller than the deflection angle of the flip bucket, and deemed that the decrease of the take-off angle is related to the relative deflector height and the approach flow Froude number for a triangular-shaped flip bucket, while it is related to the relative flow depth for a circular flip bucket. Earlier, Wu and Ruan[7] presented an improved method for calculating the lower take-off angle with consideration of the transverse fluctuating velocity and the flow depth for the aerator device with a ramp.

The trajectory of a mass point can be expressed in cases without air resistance (Fig.1) as

* Project supported by the National Natural Science Foundation of China (Grant Nos. 51509128, 51179056) the PAPD (Grant No. 3014-SYS1401).
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$$x = \frac{1}{g}(v^2 \cos\alpha \sin\alpha + v\cos\alpha\sqrt{2gE_{\rm H} + v^2 \sin^2\alpha}) \quad (1)$$

where $g = 9.8$ m^2/s, is the gravitational acceleration, $E_{\rm H}$ is the elevation difference between the approach flow and tailwater channels, α is the deflection angle, v is the (upper or lower) take-off velocity of the flow. In Fig.1, $H_{\rm o}$ is the acting water head, $h_{\rm o}$ and $v_{\rm o}$ are the approach flow depth and average velocity, respectively, resulting in the approach flow Froude number $Fr_{\rm o} = v_{\rm o}/(gh_{\rm o})^{0.5}$, w is the deflector height, $v_{\rm U}$ and $v_{\rm L}$ are the upper and lower take-off velocities of the flow at the edge of the flip bucket, and $x_{\rm U}$ and $x_{\rm L}$ are the impact points of the upper and lower trajectories onto the tailwater channel, respectively, which are determined visually from the channel side, by extending the jet trajectories.

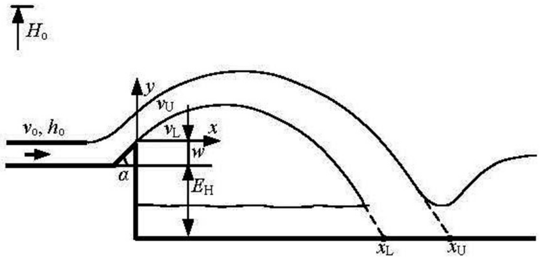

Fig.1 Definition of plane ski-jump

From Eq.(1), the impact point is dominated by v, α and $E_{\rm H}$. For a given project, however, $E_{\rm H}$ keeps constant. According to the recent investigation, there is the difference between the take-off and deflection angles for the upper or lower trajectory[5,6]. If the effect of the angle difference on the jet trajectory is small and then it could be neglected, the impact point could be conveniently estimated by means of Eq.(1) with suitable estimations of the upper or lower take-off velocity.

The objectives of this paper are to theoretically determine the effects of the take-off velocity or the deflection angle on the trajectory through a sensitivity analysis method under the condition of constant $E_{\rm H}$, to give a take-off velocity expression of the upper or lower trajectory, and to check the error range and reliability of the present estimation method for the trajectory.

With Eq.(1) giving the relation between the impact point (x) and the take-off velocity ($v_{\rm U}$ or $v_{\rm L}$) or the deflection angle (α), the sensitivity functions S_v and S_α about the take-off velocity and the deflection angle could be written as[8]

$$S_v = \frac{\left|\frac{\Delta x}{x}\right|}{\left|\frac{\Delta v}{v}\right|} = \frac{\left|\frac{\Delta x}{\Delta v}\right|}{\left|\frac{x}{v}\right|} = f'(v)\left|\frac{v}{x}\right| \quad (2)$$

$$S_\alpha = \frac{\left|\frac{\Delta x}{x}\right|}{\left|\frac{\Delta \alpha}{\alpha}\right|} = \frac{\left|\frac{\Delta x}{\Delta \alpha}\right|}{\left|\frac{x}{\alpha}\right|} = f'(\alpha)\left|\frac{\alpha}{x}\right| \quad (3)$$

respectively. And

$$f'(v) = \frac{\partial x}{\partial v} = \frac{1}{g}\left(v\sin 2\alpha + \cos\alpha\sqrt{2gE_{\rm H} + v^2\sin^2\alpha} + \frac{v^2\cos\alpha\sin^2\alpha}{\sqrt{2gE_{\rm H} + v^2\sin^2\alpha}}\right) \quad (4)$$

$$f'(\alpha) = \frac{\partial x}{\partial \alpha} = \frac{1}{g}\left(v^2\cos 2\alpha - v\sin\alpha\sqrt{2gE_{\rm H} + v^2\sin^2\alpha} + \frac{v^3\cos^2\alpha\sin\alpha}{\sqrt{2gE_{\rm H} + v^2\sin^2\alpha}}\right) \quad (5)$$

Calculations demonstrate that S_v is between 1.09 and 1.93, and S_α is between 0.02 and 0.21, when $v =$ 0–40 m/s and $\alpha = 0^{\rm o}$–$45^{\rm o}$ on the basis of Eqs.(2)-(5). Clearly, the minimum S_v is much larger than S_α in the present range. It means that the difference between the take-off and deflection angles could be neglected in the estimation of the trajectory due to the smaller effect of the angle than that of the velocity. With this knowledge, we could directly estimate the impact point of the ski-jump when the take-off velocity of the upper or lower trajectory is obtained.

Table 1 Cases and geometric parameters of models

Cases	w/m	α/$^{\rm o}$	Remarks
M12	0.01	25	M12-M32, effect of w, M21-M23, effect of α
M22	0.03	25	
M32	0.05	25	
M21	0.03	10	
M23	0.03	40	

Table 1 lists the cases and geometric parameters of the models. The cases are divided into two sets. Cases M12-M32 are used to study the effects of w, while cases M21-M23 are for the effects of α. In the