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FINITE ELEMENT NUMERICAL SIMULATION OF LAND SUBSIDENCE AND GROUNDWATER EXPLOITATION BASED ON VISCO-ELASTIC-PLASTIC BIOT'S CONSOLIDATION THEORY*

LUO Zu-jiang, ZENG Feng
College of Earth Science and Engineering, Hohai University, Nanjing 210098, China,
E-mail: luozujiang@sina.com

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Abstract: The land subsidence due to groundwater exploitation has an obvious hysteretic nature with respect to the decrease of the under groundwater level, and the uneven settlement often causes ground fissures. To study these important features, a visco-elastic-plastic constitutive relationship with consideration of the coupling of seepage and soil deformation is proposed, and a finite element model with variable coefficients based on the Biot's consolidation theory is built. With the groundwater exploitation and the land subsidence control in Cangzhou City, Hebei Province as an example, the variations of the under groundwater level and the development of the land subsidence due to the groundwater exploitation are simulated and ground fissures are predicted by the horizontal displacement calculation. The results show that the lag time between the land subsidence and the under groundwater level descent is about a month, and the simulated results of fissures agree well with the observed data. The model can well reveal the characterization of the interaction between the land subsidence and the groundwater exploitation.

Key words: hysteretic property, land subsidence, visco-elastic-plastic constitutive relationship, finite element model

Introduction

Since the eighties of the last century, the land subsidence due to the groundwater exploitation becomes more and more a research issue. During the process of consolidation, the soil shows apparent hysteretic property[1] because of time effect in the process of the clay compression. It is why the land subsidence always occurs later than the groundwater level descent. Moreover, the ground fissures caused by the groundwater exploitation go always hand in hand with the land subsidence in the temporal and spatial distributions and show a synchronism in activities and aggravations. This kind of symbiosis and consistency has the same source: the water head is decreased due to groundwater exploitation[2].

The models describing land subsidence due to groundwater exploitation are all well based on theoretical considerations. For example, in 1989, China's first model of land subsidence was built in Shanghai[3]. Afterward, the third model was built in the late 20th century and now is still under study with respect to nonlinear deformation[4]. Tianjin in 1995 built a first coupled model of land subsidence of three-dimensional seepage and one-dimensional consolidation[5]. However, because of the complexity of the problem, the rheological characteristics of aquifer and horizontal displacement were not considered in these models[6]. And constant hydraulic parameters were often assumed with respect to the permeability that varies with soil deformations, which would make great differences for practical deformation features[7]. So they can not be used to simulate the hysteretic property and more studies and improvements are required.

Therefore, in the study of land subsidence and ground fissures, one should take full account of the interaction between rheological features of soils and seepage[8]. To do so, a three-dimensional coupling model, based on the Biot's consolidation theory, is built, then according to the visco-elastic-plastic constitutive relationship and the rheological theory, the dynamic responses of the permeability coefficient and the rheological factors are considered. The comparison of calculated and observed results in the model of Cangzhou city shows that, the lag time of the land subsidence with respect to the water level descent is about one month and the intensive areas of the horizontal displacement are the high risk areas of ground fissures, The simulated results agree with the observed data.

* **Biography:** LUO Zu-jiang (1964-), Male, Ph. D., Professor

1. Biot's consolidation theory

Biot's consolidation theory, widely used in the soil mechanics studies, is a coupling theory for interactions between solid and fluid[9]. However, in the early Biot's consolidation theory, the soils were regarded as elastic bodies with voids, without considering their rheological features. The laminar pore water was coupled with solid by contractive and continuous conditions. Though some progress was made as compared to previous non-coupling methods, the calculation conditions did not much improved. In the 1990's, with the rapid development of computer technologies, the complex finite element equation for Biot's consolidation theory could be solved. Since then, the study of fluid-solid coupling theory focuses basically on the three-dimensional Biot's consolidation theory, with different assumptions of the stress and strain constitutive relation in the porous media (for example, some assumes that the porous media is elastic-plastic, and some assumes that it is visco-elastic, or different assumptions regarding the pore fluid, which could be multiphase fluid or single-phase fluid (sometimes called unsaturated porous media or saturated porous media).

For the three-dimensional soil, the Biot consolidation finite element equilibrium equation at any point in the soil can be expressed as

$$-[B]^{\mathrm{T}}[D][B]\{w\}+[B]^{\mathrm{T}}[M]u=\{f\} \tag{1}$$

where $\{w\}=\{w_x, w_y, w_z\}^{\mathrm{T}}$ is the displacement in the principal directions of anisotropy medium, $[D]$ is the stress-strain relationship matrix, which can be an elastic matrix, also may be a visco-elastic matrix depending on the models, $\{f\}=\{f_x, f_y, f_z\}^{\mathrm{T}}$ is the volume force in the x, y, z directions at the calculation point, and u is the excess pore water pressure, $[M]=\{1,1,1,0,0,0\}^{\mathrm{T}}$.

$$B_i=\begin{bmatrix} \frac{\partial N_i}{\partial x} & 0 & 0 \\ 0 & \frac{\partial N_i}{\partial y} & 0 \\ 0 & 0 & \frac{\partial N_i}{\partial z} \\ \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial x} & 0 \\ 0 & \frac{\partial N_i}{\partial z} & \frac{\partial N_i}{\partial y} \\ \frac{\partial N_i}{\partial z} & 0 & \frac{\partial N_i}{\partial x} \end{bmatrix} \quad (i=1,2,\cdots,8),$$

$$[N]=\{N_1, N_2, \cdots, N_8\}$$

is the shape function.

Moreover, according to Darcy's law and the law of conservation of energy, the finite element continuity equation of the soil is established as

$$[C]^{\mathrm{T}}\frac{\partial}{\partial t}\{w\}-\frac{1}{r_w}[C]^{\mathrm{T}}[K][C]u=\{Q\} \tag{2}$$

where

$$[C]=\begin{bmatrix} \frac{\partial N_i}{\partial x} & \frac{\partial N_i}{\partial y} & \frac{\partial N_i}{\partial z} \end{bmatrix}^{\mathrm{T}} \quad (i=1,2,\cdots,8),$$

$$[K]=\begin{bmatrix} k_{xx} & 0 & 0 \\ 0 & k_{yy} & 0 \\ 0 & 0 & k_{zz} \end{bmatrix},$$

Q is the equivalent flux array of the node, k_{xx}, k_{yy} and k_{zz} are the hydraulic conductivities in the principal directions of the anisotropy medium, respectively, r_w is the severe of water.

Equations (1) and (2) are the three-dimensional Biot consolidation equation. And the coupling mathematical model of groundwater flow and land-subsidence can be established with the initial and boundary conditions, to be solved by the finite element method[10].

2. Constitutive model of soils

The constitutive relationship concerns mechanical properties of soils, as the mathematical expression between stress-strain-strength-time. For soils, to consider the rheological properties, the deformation features are mainly reflected in the deformation time for the stress level, as a kind of visco-elastic-plastic mediums with properties of elasticity, plasticity and viscosity. The total increment of strain $\mathrm{d}\varepsilon_{ij}$ is composed of three parts: the elastic-plastic increment $\mathrm{d}\varepsilon_{ij}^{\varepsilon p}$, the visco-elastic increment $\mathrm{d}\varepsilon_{ij}^{vp}$ and the visco-plastic increment $\mathrm{d}\varepsilon_{ij}^{vp}$, therefore,

$$\mathrm{d}\varepsilon_{ij}=\mathrm{d}\varepsilon_{ij}^{\varepsilon p}+\mathrm{d}\varepsilon_{ij}^{ve}+\mathrm{d}\varepsilon_{ij}^{vp} \tag{3}$$

Each part is related with the stresses in the following manner:

(1) Elastic-plastic increment

According to the elastic-plastic constitutive