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Numerical benchmarking of fluid–structure interaction: An isogeometric finite element approach

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ABSTRACT

In this paper we describe and evaluate an isogeometric finite element program, IFEM-FSI, for doing coupled fluid–structure interaction simulations. We investigate the role played by employing higher polynomial orders and higher regularity for solving a well known benchmark problem for flow past a circular cylinder with an attached flexible bar at Reynolds number $Re=100$. Furthermore, we investigate the sensitivity to resolution in the fluid mesh as well as stiffness distribution in the mesh movement algorithm. Mesh quality is also assessed. Our simulations indicate that quadratic and cubic spline elements give better estimation of lift, drag and displacements than linear spline elements.

1. Introduction

Interaction between fluids and marine structures like monopiles and turbine support structures results in vortices which cause problems like scouring (Breusers and Raudkivi, 1991) of the seabed and vortex-induced stresses in the structures. Various techniques (De Vos et al., 2011, 2012; Nielsen et al., 2010; De Sonneville et al., 2010) are being used to prevent the scouring phenomena ranging from the use of different materials to cover the seabed to the use of smart designs of the monopile's base to break the vortices. Most of the time these techniques depend on rigid structures. Optimizing their design is relatively simpler because state-of-the-art Computational Fluid Dynamics (CFD) codes can simulate the flow structures and hence the design effectiveness. Other methods for breaking or weakening the vortices can also be thought of; like the use of flexible structures attached to the monopiles to alter the behavior of the shedded vortices. Modelling the effectiveness of such concepts will involve coupled fluid–structure interaction simulations. The simulations are complicated by the fact that they require a dynamic mesh and an accurate representation of the deformed geometry under the influence of forces throughout the simulation time. Classical simulation methods based on linear finite elements do not represent the geometry in an exact sense and introduce errors which can be reduced only by using a very high resolution. Fortunately, a new discretization method based on isogeometric analysis appears promising in this context. Isogeometric analysis (IGA), introduced in Hughes et al. (2005), has demonstrated that much can be gained in this respect by replacing the traditional low-order finite elements (FE) by volumetric NURBS (Non-Uniform Rational B-Splines). Spline approximations have some desirable properties both with respect to geometrical representation and analysis, since both the order and the smoothness of the basis functions can be easily changed. In particular, numerical results indicate that increased continuity of the finite element basis improve the approximation of both material stresses in structural analysis and sharp boundary layers in CFD analysis (Akkerman et al., 2008). An overview of recent developments in the field of isogeometric analysis can be found in Nguyen et al. (2015).

In this paper we apply the isogeometric finite element approach to the well known FSI benchmark problem presented in Turek and Hron (2006) and Turek et al. (2010) as a first step to demonstrating its benefits. The paper starts with a mathematical description of the method in solving fluid structure interaction problems. The description includes the fluid and structure solvers based on mass and momentum conservation equations. The equations are solved on a dynamic mesh to accommodate the geometry changes so a mesh mover is explained in the same section. Finally the set-up is described followed by results and conclusions.

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2. Theory

Here we present the theory behind our fluid and structural solvers, along with details of how they are coupled and how the mesh movement is done. Mesh generation is also explained. The fluid domain, consisting of an incompressible Newtonian fluid, is denoted Ω^f, while the structural domain, consisting of an elastic solid, is denoted Ω^s.

2.1. Fluid solver

The flow is mathematically described by the incompressible Navier–Stokes equations which read

$$\rho\frac{\partial \boldsymbol{u}^f}{\partial t} + \rho\left(\boldsymbol{u}^f\cdot\nabla\right)\boldsymbol{u}^f - \nabla\cdot\boldsymbol{\sigma}\left(\boldsymbol{u}^f, p\right) = \rho\boldsymbol{f} \quad \text{in } \Omega^f \nabla\cdot\boldsymbol{u}^f = 0 \quad \text{in } \Omega^f. \tag{1}$$

In this setting $\Omega \in \mathbb{R}^d$, $d=2,3$, is a suitable, sufficiently regular and open domain, ρ is the constant fluid density, p is the pressure, $\boldsymbol{u}^f$ is the fluid velocity vector and $\boldsymbol{f}$ is a volumetric body force. The Cauchy stress tensor can be written as

$$\boldsymbol{\sigma}\left(\boldsymbol{u}^f, p\right) = -p\boldsymbol{I} + 2\mu\boldsymbol{\epsilon}\left(\boldsymbol{u}^f\right),$$

where $\boldsymbol{I}$ is the identity tensor, μ the dynamic viscosity and the strain rate $\boldsymbol{\epsilon}$ is defined asFurthermore we define the boundary to be $\partial\Omega^f = \Gamma^f = \Gamma_D^f \cup \Gamma_N^f \cup \Gamma_M^f$ in order to handle boundaries with Dirichlet, Neumann or mixed boundary conditions. We denote Γ_D^f the boundaries with Dirichlet conditions, Γ_N^f the boundaries with Neumann conditions and Γ_M^f the boundaries with mixed conditions. Mixed boundary conditions are used in situations where the normal velocity component is given, usually zero, together with the tangential stresses, and can model symmetry planes and slip or friction conditions.

The variational formulation can now be expressed as follows: Find $(\boldsymbol{u}^f, p) \in \boldsymbol{U}\times Q$ such that

$$\left(\rho\frac{\partial \boldsymbol{u}^f}{\partial t}, \boldsymbol{v}\right) + c\left(\boldsymbol{u}^f; \boldsymbol{u}^f, \boldsymbol{v}\right) + b(p, \boldsymbol{v}) + a\left(\boldsymbol{u}^f, \boldsymbol{u}^f\right) + b\left(q, \boldsymbol{u}^f\right) = f(\boldsymbol{v}) \tag{2}$$

for $(\boldsymbol{v}, q) \in \boldsymbol{V}\times Q$. We have defined the spaces

$$\begin{aligned}\boldsymbol{U} &= \boldsymbol{H}_{\Gamma_D,\Gamma_M^\perp}(\Omega) = \left\{\boldsymbol{v}\in\boldsymbol{H}^1(\Omega)|\boldsymbol{v}=\boldsymbol{u}_{\boldsymbol{D}}^f \text{ on } \Gamma_D \text{ and } \boldsymbol{v}\cdot\boldsymbol{n} = u_\perp^f \text{ on } \Gamma_M\right\}\boldsymbol{V}\\ &= \boldsymbol{H}_{\Gamma_D,\Gamma_M^\perp;0}(\Omega) = \left\{\boldsymbol{v}\in\boldsymbol{H}^1(\Omega)|\boldsymbol{v}=0 \text{ on } \Gamma_D \text{ and } \boldsymbol{v}\cdot\boldsymbol{n} = 0 \text{ on } \Gamma_M\right\}Q\\ &= L^2(\Omega),\end{aligned}$$

where $\boldsymbol{u}_{\boldsymbol{D}}^f$ and $u_\perp^f$ both are given functions and $\boldsymbol{n}$ is the unit outer normal on Γ. We have also defined the forms

$$\begin{aligned}a(\boldsymbol{u}^f, \boldsymbol{v}) &= 2\int_\Omega \mu\boldsymbol{\epsilon}(\boldsymbol{u}^f):\boldsymbol{\epsilon}(\boldsymbol{v})\,\mathrm{d}\boldsymbol{x} b(q, \boldsymbol{v}) = -\int_\Omega (\nabla\cdot\boldsymbol{v})q\,\mathrm{d}\boldsymbol{x} c(\boldsymbol{w}; \boldsymbol{u}^f, \boldsymbol{v})\\ &= \int_\Omega \rho(\boldsymbol{w}\cdot\nabla)\boldsymbol{u}^f\cdot\boldsymbol{v}\,\mathrm{d}\boldsymbol{x} f(\boldsymbol{v}) = \int_\Omega \rho\boldsymbol{f}\cdot\boldsymbol{v}\,\mathrm{d}\boldsymbol{x} + \int_{\Gamma_N}\boldsymbol{t}\cdot\boldsymbol{v}\,\mathrm{d}\boldsymbol{s},\end{aligned}$$

where $\boldsymbol{t} = \boldsymbol{\sigma}\cdot\boldsymbol{n}$ is the traction vector on Γ.

2.1.1. Isogeometric finite element approximation

In this work we employ an isogeometric finite element method similar to what was introduced in Hughes et al. (2005) and presented in Cottrell et al. (2009). The isogeometric finite element method approximates the solution by using a spline basis of polynomial order p and regularity C^{p-1}. In traditional finite element formulations C^0 Lagrange polynomials of low order (typically $p=1$ or $p=2$) are used. Our approach is based on a conforming finite element approximation, i.e.

$$\boldsymbol{U}_h \subset \boldsymbol{U}, \quad \boldsymbol{V}_h \subset \boldsymbol{V}, \quad Q_h \subset Q.$$

The discrete approximation spaces $\boldsymbol{U}_h$, $\boldsymbol{V}_h$, and Q_h are chosen as the isogeometric finite element spaces. This gives the semi-discrete formulation of the variational problem stated in Eq. (2): Find $\left(\boldsymbol{u}_h^f, p_h\right) \in \boldsymbol{U}_h\times Q_h$ such that

$$\begin{aligned}&\left(\rho\frac{\partial \boldsymbol{u}_h^f}{\partial t}, \boldsymbol{v}_h\right) + c\left(\boldsymbol{u}_h^f; \boldsymbol{u}_h^f, \boldsymbol{v}_h\right) + a\left(\boldsymbol{u}_h^f, \boldsymbol{u}_h^f\right) + b(p, \boldsymbol{v}_h)\\ &\qquad + b\left(q, \boldsymbol{u}_h^f\right) = f(\boldsymbol{v}_h)\end{aligned} \tag{3}$$

for all $(\boldsymbol{v}_h, q_h) \in \boldsymbol{V}_h\times Q_h$.

As described in Nordanger et al. (2015) we have developed a block-structured B-spline isogeometric finite element approximation of the Navier–Stokes equations described above. A domain Ω can be subdivided into a number of patches Ω_e such that $\Omega = \cup_{e=1}^N \Omega_e$, where what we call a patch is equivalent to a block. To construct a B-spline basis for Ω we associate for each patch a knot-vector in each coordinate direction

$$\Xi_k^e = \left\{\xi_{1,k}^e, \xi_{2,k}^e, \ldots, \xi_{n_k^e+p_k^e+1,k}^e\right\}$$

for $k = 1, \ldots, d$. Here, n_k^e is the number of B-spline basis functions associated with the knot span. The B-spline basis for patch Ω_e on the parametric domain $\hat{\Omega} = (0, 1)^d$ is written as $\hat{\mathcal{S}}_{\boldsymbol{\alpha}^e}^{\boldsymbol{p}^e}$ where the multi-indices $\boldsymbol{\alpha}^e = (\alpha_1^e, \ldots, \alpha_d^e)$ and $\boldsymbol{p}^e = (p_1^e, \ldots, p_d^e)$ denote the regularity and order for the basis in each coordinate direction, respectively. The corresponding basis for the physical domain Ω_e can be expressed using the coordinate mapping ϕ_e: $\hat{\Omega} \to \Omega_e$ as

$$\mathcal{S}_{\boldsymbol{\alpha}^e}^{\boldsymbol{p}^e} = \left\{v_h \middle| v_h\circ\boldsymbol{\phi}_e \in \hat{\mathcal{S}}_{\boldsymbol{\alpha}^e}^{\boldsymbol{p}^e}\right\}.$$

If the variational formulation allows a discontinuous approximation the spline finite element basis for the domain Ω can be defined as

$$\mathcal{S}_h = \left\{v_h \middle| v_{h|\Omega_e} \in \mathcal{S}_{\boldsymbol{\alpha}^e}^{\boldsymbol{p}^e}\right\}.$$

If we assume that the knot-vectors and geometrical mapping $\boldsymbol{\phi}_e$ for all the patches are consistent on common edges and faces we can define a continuous basis

$$\mathcal{S}_{\boldsymbol{h}} = \left\{v_h \in C(\Omega) \middle| v_{h|\Omega_e} \in \mathcal{S}_{\boldsymbol{\alpha}^e}^{\boldsymbol{p}^e}\right\}.$$

We use the same basis for the geometry as for the discretization of the velocity and the pressure.

2.1.2. Projection method

In order to solve the mixed variational problem given in Eq. (3) the following inf-sup condition

$$\inf_{q_h\in Q_h, q_h\neq 0}\sup_{\boldsymbol{v}_h\in\boldsymbol{V}_h, \boldsymbol{v}_h\neq 0}\frac{b(q_h, \boldsymbol{v}_h)}{\|\,q_h\,\|_{L^2(\Omega)}\|\,\boldsymbol{v}_h\,\|_{\boldsymbol{H}^1(\Omega)}} \geq C > 0.$$

needs to be satisfied in order to avoid spurious pressure modes (Brezzi, 1974). This imposes restrictions on the choices of $\boldsymbol{V}_h$ and Q_h.

Traditionally a mixed finite element method with different approximation spaces for pressure and velocity is required. In this work we use a pressure correction projection scheme which allows for equal-order approximation of the velocity and pressure. This is based on the work pioneered by Chorin (1968) and Temam (1969) in the late 1960s. In order to stabilize the equal-order approximation we employ Minev stabilization as described in Minev (2001). A backward differentiation formula of order 2 (BDF2 scheme) is used for the time integration.

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