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Diffraction of surface water waves by an undulating bed topography in the presence of vertical barrier

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ABSTRACT

In this paper, the problem of diffraction of surface water waves by an undulating bed and a vertical barrier is examined. Here, two different barrier configurations are studied, namely, (i) a partially immersed barrier and (ii) a bottom standing barrier. Perturbation analysis containing a dimensionless parameter which describe the smallness of bottom undulation is employed to find the solution of the mixed boundary value problem (BVP). This analysis reduces the BVP into two BVPs up to first order. The zeroth order BVP represents the problem of scattering of water waves by a vertical barrier. Using eigenfunction expansion method this zeroth order BVP leads to a pair of dual series relations which are solved by the application of least-squares method. The first order BVP containing the solution of the zeroth order BVP is solved by suitable application of the Green's function technique. From the solution of the first order BVP, the physical quantities, namely, the first order reflection and transmission coefficients are obtained in terms of integrals involving the shape function representing the bottom undulation and the solution of the zeroth order BVP. A patch of sinusoidal ripples which closely corresponds to some obstacles produced by nature due to ripple growth and alluviation of sand is considered for which the explicit expressions for the first order reflection and transmission coefficients are obtained. The variation of these coefficients on the barrier length, gap above or below the barrier, ripple amplitude and the number of ripples at the bottom is illustrated graphically. The other important factors of the study, namely, the hydrodynamic force and the moment are also studied and depicted graphically. Furthermore, the energy balance relation which ensures the correctness of the numerical results, is derived and verified.

1. Introduction

The problems involving the scattering of water waves by vertical barriers have been drawing a great attention of many researchers because of their engineering applications such as wavemakers and breakwaters which protect a harbor from the rough sea. Dean (1945) derived the reflection coefficients from the linearized solution of water wave scattering involving thin plane vertical barrier by using complex variable technique. Ursell (1947) used the singular integral equation approach based on Havelock's expansion to obtain the solution of the problem of water wave diffraction by thin vertical barrier partially immersed in deep water. Porter (1972) obtained the solution of the problem involving wave transmission through a gap in a vertical barrier in deep water by using the complex variable technique as well as Green's integral theorem. Losada et al. (1992) solved the problem involving scattering of water waves by thin vertical barriers for different barrier configurations by using eigenfunction expansion method. Mandal and Dolai (1994) and Porter and Evans (1995) derived the solution of the problem involving scattering of water waves by vertical barrier by using the Galerkin approximation method. Banerjea et al. (1996) utilized the one-term and multi-term Galerkin approximation to evaluate the reflection coefficient for the problem handled by Porter (1972). Mandal and Chakrabarti (1999) obtained the approximate solutions of a number of water wave scattering problems involving thin vertical barriers by utilizing the Galerkin's method. Kanoria et al. (1999) discussed the problem involving scattering of water waves by different kinds of thick vertical barriers. Sahoo et al. (2000) and Lee and Chwang (2000) discussed the problem of water wave scattering by vertical permeable barriers with the help of eigenfunction expansion method. Yip et al. (2002) discussed the trapping of surface waves by vertical porous and flexible barrier and observed that the deflection at the free end is smaller for the bottom touching barrier than for the surface-piercing barrier. Feng and Lu (2011) investigated the problem of interaction of water waves with floating structures of arbitrary shapes with the aid of an eigenfunction expansion method. Karmakar and Soares (2014)

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Nomenclature

a	Length of partially immersed barrier
b	Gap from the free surface to bottom standing barrier
B	Barier
$c(x)$	Shape of the undulation
c_0	Amplitude of the sinusoidal ripples
g	Acceleration due to gravity
G	Gap
$\widehat{G}$	Green's function
h	Depth of the fluid
λ	Wave number of the sinusoidal ripples
M	Number of ripples
R	Reflection coefficient associated with reflected wave
R_0	Zeroth order reflection coefficient
R_1	First order reflection coefficient
t	Time
T	Transmission coefficient associated with transmitted wave
T_0	Zeroth order transmission coefficient
T_1	First order transmission coefficient
ε	Small parameter giving measure of the undulation
σ	Frequency of the incident wave
ϕ	Complex-valued velocity potential
ϕ_0	Zeroth order complex-valued velocity potential
ϕ_1	First order complex-valued velocity potential
k_0	Wave number

investigated the linear water wave propagation of oblique surface waves through multiple bottom standing flexible porous barriers.

The problems involving the interaction of surface waves with irregular bottom topography of a seabed are also interesting for many investigations due to their application in marine and coastal engineering. Among many investigations, one of them is to study the effect of naturally occurring obstacle such as sand ripples on wave propagation. Miles (1981) derived the first order reflection and transmission coefficients in terms of integrals by using perturbation theory along with the Finite cosine transformation technique when oblique waves are incident to a cylindrical obstacle. Davies (1982) and Davies and Heathershaw (1984) discussed the problem of the reflection of the incident waves by irregular bottom using Fourier transform technique. Mandal and Basu (1990) generalized the Miles problem with the inclusion of surface tension at the free surface. Martha and Bora (2007) and Bora and Martha (2008) considered the problem of water wave scattering by different types of uneven sea-bed by making use of perturbation theory. Mandal and Gayen (2006) analyzed the scattering of surface water waves in single-layer fluid using multi-term Galerkin approximation and Green's integral theorem. Das and Bora (2014a) considered the problem of wave reflection by a vertical rectangular porous structure placed on a elevated bottom. Das and Bora (2014b) studied the reflection of surface water waves by placing the vertical porous structure on a multi-step impermeable horizontal bottom.

However, in the present situation, the thin vertical barriers along with the irregular bottom topography will serve as an effective breakwater for coastal engineering. Thus, it is an endeavor to consider the problems of multiple scattering, which facilitate the real and practical situations. In the present paper, the barrier is rigid and fixed, which may be one of the two types whose positions are described in the next section. A mixed boundary value problem occurs in a natural way while studying this scattering problem mathematically. Employing perturbation analysis involving the small dimensionless parameter ε which characterizes the smallness of bottom undulation, the original boundary value problem is reduced to two boundary value problems, namely, BVP-I and BVP-II. The BVP-I is obtained by equating the coefficients of the powers of ε^0 where as BVP-II is obtained by equating the coefficients of the powers of ε. The BVP-I corresponds to the problem of scattering of water waves by a vertical barrier in water of uniform finite depth and this is solved with the aid of eigenfunction expansion method which leads to an over-determined system of linear equations. The solution of such over-determined system is obtained with the application of least-squares method and the numerical values of the zeroth order reflection and transmission coefficients are determined. The BVP-II, which involves the solution of the zeroth order BVP, represents the radiation problem in water of uniform finite depth and this is solved with the aid of the Green's function technique. With the help of the solution of the first order BVP, the first order reflection and transmission coefficients are obtained in terms of the integrals involving the shape function $c(x)$ and the solution of the zeroth order BVP. A special form of the bottom undulation representing a patch of sinusoidal ripples which is symmetric about the plane of the barrier is considered to evaluate these reflection and transmission coefficients. For this particular example, the numerical values of the first order reflection coefficient are obtained and depicted graphically. The effect of the various parameters such as the length of the barrier, length of the gap, the number of ripples, the amplitudes of the ripples on the first order reflection coefficient is examined in detail. Furthermore, the hydrodynamic force and moment are also investigated. The behavior of the hydrodynamic force and moment on the vertical barrier in the presence of bottom undulation is also studied and shown through different graphs. In the present paper, an effort has been made to find the solution of BVP-II, force, moment and energy balance relation which are the main factors among many investigations.

2. Mathematical formulation

A two dimensional Cartesian coordinate system is chosen in which the x-axis represents the position of undisturbed free surface and the y-axis is taken positive vertically downwards into the fluid. The bottom has small undulation and is described by $y = h + \varepsilon c(x)$ where $c(x)$ is a function with compact support describes the shape of the bottom undulation and $c(x) \to 0$ as $|x| \to \infty$, h denotes the uniform finite depth of the fluid far to either side of the undulation of the bottom and the non-dimensional number $\varepsilon(\ll 1)$ gives the measure of the smallness of the bottom undulation. Consider a thin rigid vertical barrier denoted by B, is fixed in the fluid, whose position (see Fig. 1) is described as

(a) Case I: $x = 0, y \in B = [0, a]$ (a partially immersed barrier),
(b) Case II: $x = 0, y \in B = [b, h + \varepsilon c(0)]$ (a bottom standing barrier).

2.1. Governing equations

It is assumed that the fluid is inviscid, incompressible and the motion to be irrotational and simple harmonic in time. Under the assumption of linearized theory of water waves, the complex-valued potential function $\phi(x, y)$ describing small motion in water satisfies Laplace's equation