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Coherent structures in phase space, governing the nonlinear surge motions of ships in steep waves

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ABSTRACT

In steep multi-chromatic seas, ship surge dynamics can become intricate and the full variety of exhibited motions is unknown. This accrues, partly, from the nonlinear nature of surge motion; and partly because, for multi-frequency waves, the phase-space flow of the dynamical system becomes time-dependent. Accordingly, conventional concepts that were applied in the past for analyzing stationary phase-space flows are rendered incapable to support in-depth exploration of ship dynamics. Towards overcoming this limitation, use of the concept of hyperbolic *Lagrangian Coherent Structures* (LCSs) is proposed. These phase-space objects can be regarded as the “finite-time” generalizations of the stable and unstable manifolds of hyperbolic fixed points defined in “time-invariant” dynamical systems. They can be described as, locally, the strongest repelling or attracting material surfaces (curves in the case of 2-dimensional systems) advected with the phase flow. We have identified hyperbolic LCSs that are innate to the phase-flow associated with the surge motion of a ship in astern seas. To the global approach of LCS identification, a supplementary computational scheme is incorporated, aiming to track, in space-time, local “features” of the flow, connected with surf-riding. The emerging toolset can enhance current efforts towards a rigorous assessment of ship dynamic stability in following seas.

1. Introduction

The mechanisms generating surf-riding behaviour for a ship operating in regular following seas have been extensively studied in the past (e.g. Kan, 1990; Spyrou, 1996). However, gaining understanding beyond the context of harmonic waves has been considered daunting; because the multi-frequency wave field brings-in new qualitative features in ship response, by-and-large as yet unrecognised, descending from the time-dependent nature of the system's phase flow.

For “regular sea” scenarios, it is well known that surf-riding can be identified as an equilibrium solution of the surge equation of motion. Such solutions may appear in coexistence with the ordinary periodic type of ship surge response, or they may even entirely dominate the surge behaviour of a ship. A detailed account of progress on understanding the nonlinear surging and surf-riding of a ship in harmonic waves can be found in Spyrou (2006). Consideration, though, of more general wave forms introduces profound complications. For irregular seas, the key description of surf-riding as a stationary state needs to be reappraised, since one cannot reasonably assume that the underlying non-autonomous dynamical system will admit constant solutions. Hence, a broader definition of surf-riding is entailed.

With these difficulties recognized, a phenomenological approach to surf-riding in irregular seas has been proposed recently, expanding upon the notion of wave celerity and its role in signalling a ship's capture to surf-riding (Spyrou et al., 2012, 2014a). In particular, definition and methods for the calculation of wave celerity for an irregular seaway were proposed and their relevance to the problem of surf-riding was evaluated. The appeal of such an approach is that it enables a straightforward statistical approximation of the probability of surf-riding in irregular seas, by setting up a direct counting scheme of velocity threshold exceedances.

In another recent work, Belenky et al. (2012) endeavoured to gain insight into the surge dynamics in multi-chromatic following waves through the identification of the points of the wave profile where, equilibrium of forces along ship's longitudinal direction is instantaneously satisfied. For the calculation of such points, celerity of irregular waves, whereabout ship's position, must be known. This technique could be useful, in some instances, as an approximate calculation scheme.

Also, Spyrou et al. (2014b) examined the possibility of extracting and tracking “features” related to the surge dynamics in irregular seas. The word “feature” is attached here to any object ostensibly relevant to the realisation of surf-riding. It was discovered that, such features can be identified among the elements

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of the zero set of the "acceleration field" i.e., points of the phase-plane where the acceleration and its time derivative attain, instantly, zero values.

In the current work, methods having potential to yield insight into the dynamics of the surge motion in multi-chromatic astern seas are employed. In particular, the concept of hyperbolic Lagrangian Coherent Structures is applied and its capacity in unveiling the changing-in-time organization of system's phase flow is evaluated. Through their organizing role, these structures can be considered as analogues of the stable and unstable manifolds of hyperbolic fixed points defined in autonomous dynamical systems. For their identification, one can choose among a number of different methods. Here, a popular numerical scheme is selected, based on the calculation of the spatial distribution of the largest finite-time Lyapunov exponent. Furthermore, a scheme aiming to the tracking of critical points of the vector field, defined by the value of acceleration along the surge direction and of its material derivative, is applied. This supplementary numerical scheme is based on the Feature Flow Field concept, which addresses the problem of feature tracking in non-stationary flow fields (Theisel and Seidel, 2003).

With the current work it is aimed to propose concepts and toolsets which can enable deeper understanding of the surf-riding and broaching-to behaviour in irregular seas. Such phenomena of extreme ship behaviour currently receive detailed attention at IMO and, it is expected that they will be covered in future legislation as additional dynamic stability requirements (Peters et al., 2011).

2. Concepts and computational tools

2.1. Lagrangian Coherent Structures

The concept of Lagrangian Coherent Structures seems to have emerged as result of the interbreeding of ideas originating from the fields of dynamical systems theory and fluid dynamics. Although the term was firstly introduced by Haller and Yuan (2000), many people have contributed in the development of computational strategies for their identification – for a short review see Shadden (2011). LCSs have been extensively used during the last years in a wide range of applications concerning physical and biological flows, while the theory, as well as efficient calculation methods, are still developing.

Although one can select among different schemes for the identification of hyperbolic LCSs [such as the finite-size Lyapunov Exponent (FSLE) approach, or the variational theory of hyperbolic LCSs developed recently by Haller (2011) that enables a more rigorous computation] for the needs of the current study we will consider a widely used computational procedure involving the calculation of the largest finite-time Lyapunov exponent (FTLE) field.

Let us consider the following dynamical system that defines a flow on the plane,

$$\dot{x} = f(x, t), \quad x \in D \subset \mathbb{R}^2, t \in \left[t^-, t^+\right] \subset \mathbb{R} \tag{1}$$

A trajectory of system (1) at time t, starting from the initial condition x_0 at t_0, will be denoted by $x(t;\ t_0,\ x_0)$. We can write for the flow map $F_{t_0}^t(x_0)$ of (1),

$$F_{t_0}^t : \quad D \to D$$

$$x_0 \mapsto x(t; t_0, x_0) \tag{2}$$

Through (2), the phase-particle passing from x_0 at time t_0 is associated with its position at time t. We, furthermore, consider two infinitesimally close phase-particles, located at x_0 and $x_0 + \delta_0$ at time t_0. The magnitude of the linearized perturbation at time $t_0 + \tau$ is given by (see, e.g. Shadden (2011)),

$$\|\delta\| = \|\nabla F_{t_0}^{t_0+\tau}(x_0)\delta_0\| = \|\delta_0\| \sqrt{e_0^{\mathrm{T}} \left[\nabla F_{t_0}^{t_0+\tau}(x_0)\right]^{\mathrm{T}} \nabla F_{t_0}^{t_0+\tau}(x_0) e_0} \tag{3}$$

In the above, e_0 is the unit vector along the direction of δ_0, A^{T} denotes the transpose of A, while $\nabla F_{t_0}^{t_0+\tau}(x_0)$ is the deformation gradient and $C_{t_0}^{t_0+\tau}(x_0) = \left[\nabla F_{t_0}^{t_0+\tau}(x_0)\right]^{\mathrm{T}} \nabla F_{t_0}^{t_0+\tau}(x_0)$ is the right Cauchy–Green deformation tensor, both evaluated at x_0. $C_{t_0}^{t_0+\tau}(x_0)$ is a real symmetric, positive definite tensor and, as such, it has real positive eigenvalues, $\lambda_i, i = 1, 2$. Moreover, the corresponding eigenvectors, e_i, $i = 1, 2$, form an orthonormal basis.

The Cauchy–Green deformation tensor provides a measure of how line elements in the neighbourhood of x_0 deform under the flow; i.e., how the lengths and the angles between line elements change, when considering the configuration in the close vicinity of $x(t; t_0, x_0)$ at times t_0 and $t_0 + \tau$. A circular blob of initial conditions centred at x_0 will evolve into an ellipse, with the major (minor) axis aligned with the direction of the eigenvector e_2 (e_1). The coefficients of expansion along these directions will be given by $\sqrt{\lambda_i}$, $i = 1, 2$.

The finite-time Lyapunov exponents are defined as follows,

$$\Lambda_i = \frac{1}{|\tau|} \ln \sqrt{\lambda_i}, \qquad i = 1, 2 \tag{4}$$

The largest FTLE, Λ_2, is usually referred to as "FTLE" without distinction. By virtue of (4), Λ_2 can be regarded as a time-averaged measure of stretching and therefore, as a (rough) measure of a trajectory's hyperbolicity. Yet, as noted by Haller (2011) and Shadden (2011), this does not hold in general.

Through the calculation of the spatial FTLE distribution, the identification of LCSs is made possible. The latter will appear as local maximizing curves of the FTLE field. Typically, the calculation of the field is performed on the basis of a structured grid of initial conditions spanning a considered domain at a given time t_0. The grid is integrated over a specified time interval τ, using a numerical integration algorithm. Once the final position of each grid point is calculated, the deformation gradient is obtained by implementing a finite difference scheme on the nodes of the initial grid. In the final step of the procedure, the largest eigenvalue of the deformation gradient is computed and the FTLE field is calculated directly from expression (4). The location of repelling/attracting LCSs can be identified as ridges of the FTLE field when forward/backward integration times are considered.

2.2. Feature tracking

According to Spyrou et al. (2014b), "features" relevant to the problem of surf-riding in multi-chromatic waves can be identified on the basis of critical points of a planar vector field, with coordinates that correspond to the acceleration, along the surge direction, and its material derivative. The paths of such features can be calculated using the Feature Flow Field (FFF) method. With respect to the latter, and given a vector valued function of the form,

$$a(x_1, x_2, t) = (a_1(x_1, x_2, t), a_2(x_1, x_2, t)) \tag{5}$$

a three-dimensional vector field $w(x_1, x_2, t)$ is constructed, by demanding that vector w points toward the direction of minimal change of a in a first order approximation. This direction is defined by the intersection of the planes perpendicular to ∇a_1 and ∇a_2, where the ∇ operator is related to the three-dimensional Euclidian space with coordinates (x_1, x_2, t) (Theisel and Seidel, 2003). Thus,

$$w = \nabla a_1 \times \nabla a_2 \tag{6}$$