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# A novel Wake Oscillator Model for simulation of cross-flow vortex induced vibrations of a circular cylinder close to a plane boundary

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ABSTRACT

This paper presents a novel Wake Oscillator Model for predicting Vortex-Induced Vibration (VIV) of a circular cylinder close to a plane boundary. The innovation lies in its extension of the classic Van der Pol equation by introducing an additional nonlinear term and two empirical coefficients to account explicitly for the effects of the gap ratio and mass ratio. The model parameters are calibrated against available experimental data and a number of validation tests are then performed. Despite the intrinsic limitations of the Wake Oscillator Model approach, the simulation results demonstrate that the predicted variations of vibration amplitudes with the reduced velocity are broadly consistent with the experimental data and the model is also capable of capturing the different vortex shedding modes. With further refinements, the proposed modelling approach is expected to provide an efficient engineering means to predict both the overall variation trend and maximum value of vibration amplitude of a circular cylinder close to a plane boundary over the full range of reduced velocities.



## 1. Introduction

Pipelines are widely used in the offshore and subsea engineering to transport oil and gas. Due to uneven sea bed terrain or local scour, free spans are very common, and the length of a free span can even reach 100 m. Subjected to the ocean current or wave, these pipelines can suffer from time varying stresses and fatigue damage resulting from the vortex induced vibrations (VIV). Therefore, determination of allowable free span length plays a crucial role in design of offshore pipelines, which requires the prediction of the VIV characteristics of a pipeline with various span.

In the past decades, a number of experimental studies have been carried out to understand the effect of mass, damping, surface roughness and Reynolds numbers on the dynamic response of VIV of cylindrical structures. These include flume experiments performed by Feng (1968), Khalak and Williamson (1996), Raghavan and Bernitsas (2008) and Lee and Bernitsas (2011). Similarly numerous theoretical analysis of flow vortex and flow-structure interactions have also been performed using various numerical methods, such as discrete vortex model (Sarpkaya, 1989; Blevins, 1991), spectral element technique (Newman and Karniadakis, 1997; Blackburn and Henderson, 1996), the vortex blob method (Hall and Griffin, 1993; Shiels, 1998) and LES simulations (Pontaza and Chen, 2007). Despite the proven capability of Computational Fluid Dynamics (CFD) to predict the dynamic characters of VIV, such methods are limited by their extensive computational requirements for simulations at realistic Reynolds numbers (Wu et al., 2012). In order to obtain approximate solutions at a much lower computational cost, various Wake Oscillator Models based on semi-empirical methods are also proposed. For example, on the basis of theoretical considerations on the flow momentum and the Van der Pol type differential equation, Iwan and Blevins (1974) simulated the lift force on the cylinder, while in the Milan Wake Oscillator Model developed by Falco et al. (1999), the effects of VIV were studied with a fictitious vibrating mass attached to the node. All Wake Oscillator Models including those described above seek to solve an ordinary differential equation governing a flow variable $q$ that is introduced to represent the oscillation of the flow wake behind a rigid cylinder rather than reproduce the physics of the fluid-structure interaction in detail by solving the full fluid dynamic equations. By applying the lift force determined from $q$, the vibration equation of the cylinder can be solved to determine its vibration characteristics.

Recent progress including many new insights on the dynamics of Wake Oscillator Models for single degree of freedom VIV can be found in Facchinetti et al. (2004) while, Ogink and Metrikine (2010), also provided a thorough review of the topic including models for system with two degrees of freedom.

However, with the pipeline being close to seabed, the effect of

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the gap between the pipeline and the seabed on the dynamic response of the pipeline needs to be considered as these effects have been shown to be significant in a number of experiments, such as Fredsoe et al. (1987), Tsahalis and Jones (1981) and Yang et al. (2009). These researchers found that for a given flow and pipe diameter, the VIV are essentially determined by the relative gap size and unit mass of the cylinder. To investigate the vortex shedding past a circular cylinder near a wall, Lei et al. (2000) solved the Navier–Stokes equations and the pressure Poisson equation for two-dimensional time-dependent viscous flows using a finite difference method in a curvilinear coordinate system. Based on a two-dimensional standard high Reynolds number k–ε turbulence model, Ong et al. (2010) computed high Reynolds number flows around a circular cylinder close to a flat seabed. More recently, on the basis of Reynolds-Averaged Navier–Stokes (RANS) equations, Zhao and Cheng (2011) applied the Arbitrary Lagrangian Eulerian (ALE) scheme to deal with the effect of the boundary and analyse the influence of gaps on the VIV. Nevertheless, all of these calculations are rather complicated and time consuming. There are also considerable discrepancies between numerical predictions and experimental data.

In this paper, the classic Wake Oscillator Model based on the Van der Pol equation is extended to account for the additional nonlinear flow effects originated from pipeline-seabed interaction. For simplicity, the tension variation along the pipeline is ignored. Although the response of real free span pipelines take place in both in-line and cross-flow directions, the model development and analysis are restricted to a 1D problem and focused on the simulation of cross-flow vortex induced vibrations of a circular cylinder close to a plane boundary. Compared with the classic Wake Oscillator Model, the model developed here has one additional nonlinear term and two empirical coefficients, which are calibrated against the available experimental data. The performance of the model is assessed and discussed.

## 2. Proposed Wake Oscillator Model

### 2.1. Vibration model of the cylinder

Transverse oscillation of an elastically supported rigid circular cylinder of diameter $D$, subjected to a stationary and uniform flow of free stream velocity $U$, is as depicted in Fig. 1.

The general structure response equation in a plane cross-flow is usually formulated in terms of a linear oscillator variable q as given by Facchinetti et al. (2004) as

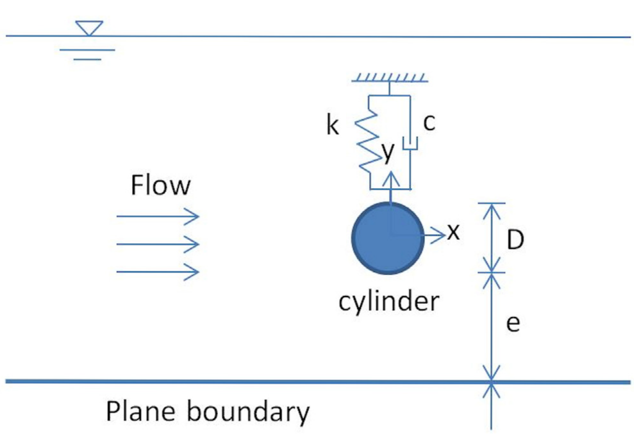


**Fig. 1.** Spring supported cylinder close to a plane boundary subjected to current.

**Table 1**
Physical quantity relative to dynamic response of pipeline near the seabed.

| Physical quantities | Symbol | Dimension |
|---|---|---|
| ***Ocean current*** | | |
| Fluid mass density | $\rho$ | $ML^{-3}$ |
| Undisturbed flow velocity | $U$ | $LT^{-1}$ |
| Strouhal number | $S_t$ | 1 |
| ***Pipeline*** | | |
| Diameter | $D$ | L |
| Mass per metre | $m$ | $ML^{-1}$ |
| Natural frequency | $f_n$ | $T^{-1}$ |
| Gap between pipe and seabed | $e$ | L |

$$m_e\ddot{y}+\left(c+\gamma\rho D^2\Omega_f\right)\dot{y}+ky=\frac{1}{4}\rho U^2DC_{L0}q. \tag{1}$$

where $(\bullet)$ means derivative with respect to time $t$, $y$ is the displacement of the cylinder in the cross-flow direction, $\Omega_f$is the vortex-shedding angular frequency, $\Omega_f=2\pi S_t U/D$, the mass for unit length of the cylinder, $m_e$ is taken into account both the mass of structure $m$ and the fluid-added mass $m_f=mC_a$, in which $C_a$ is the added mass coefficient, $\rho$ is the density of the fluid, $U$ is the flow velocity in the free stream, $D$ is the diameter of the cylinder, $k$ is a spring constant and $c$ is the structure damp $c = 2m_e\xi\Omega_s$ and $\xi$ is the reduced damping, $\Omega_s$ is the structural angular frequency $\Omega_s=(k/m_e)^{0.5}$. Pantazopoulos (1994) presented empirical formulas to calculate the damping of flow $\gamma = C_D/4\pi S_t$ and related the drag coefficient $C_D=C_{D0}(1 + 2y/D)S_tU_r$, in which $C_{D0}$ is reference drag coefficient, and the reduced flow velocity $U_r=U/f_nD$. $q = 2C_L/C_{L0}$, is a flow variable that is commonly referred to as a reduced vortex lift coefficient in which $C_L$ is the lift coefficient and $C_{L0}$ is the reference lift coefficient, usually taken as a constant of 0.3.

The physical quantities relevant to the calculation of dynamic response of a circular cylinder close to a plane boundary are listed in Table 1.

### 2.2. Modified Wake Oscillator Model

The standard Van der Pol equation for simulating the fluid wake effect around a cylinder without the influence of a boundary is

$$\ddot{q} + \epsilon\Omega_f\left(q^2-1\right)\dot{q}+\Omega_f^2\ q=F. \tag{2}$$

where the dot represents time derivative, $F$ is assumed to be related to the acceleration of the displacement, $F = A/D\ddot{y}$ (Facchinetti et al., 2004), and $\epsilon$ and $A$ are parameters which need to be determined empirically and have typical values $\epsilon = 0.3$ and $A = 12$.

When a cylinder is located very close to the seabed, the flow wake generated behind the cylinder can be affected so significantly that its motions cannot be described adequately with the existing Wake Oscillator Models. Considering the heuristic nature of the wake oscillator formulation, it is expected that the predictive capability of the original Wake Oscillator Model such as Eq. (2) may be extended by adding additional terms to account for the influence of gap size (or relative gap size) and the mass of the cylinder. On the basis of the experimental results of Tsahalis and Jones (1981), Fredsoe et al. (1987) and Yang et al. (2009) and extensive trials of various model formulations, a new model equation for the wake effect is proposed in the form of Eq. (3).

$$\ddot{q}+\epsilon\Omega_f\left(q^2-1\right)\dot{q}+\Omega_f{}^2q+\alpha\left(m^*,\ e/D\right)f_n\Omega_fq^2=\beta\left(m^*,\ e/D\right)A/D\ddot{y} \tag{3}$$

in which the two parameters associated with the fourth and fifth terms, $\alpha$ and $\beta$, are considered to be the functions of mass ratio $m^*$