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Free transverse vibration of ocean tower

Ankit*, Nabanita Datta

Department of Ocean Engineering and Naval Architecture, Indian Institute of Technology, Kharagpur, West Bengal 721302, India

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ABSTRACT

This paper studies a continuous, elastic model of an ocean tower, partially submerged in water, undergoing free transverse vibration in a plane. The tower is modeled as a non-uniform Timoshenko beam which is supported by an eccentric tip mass on one end and a non-classical damped foundation on the other. The foundation is modeled as a combination of translational and rotational springs and dampers. The effect of shear deformation and rotary inertia is included in the analysis. The free vibration equation is derived using Hamilton's variational principle based on two approaches, Rayleigh Ritz Method (RRM) and Finite Element Method (FEM), which show a good agreement in results. The computational efficiency of RRM over FEM is shown using a convergence study. Finally, a parametric study is done to demonstrate the dependence of natural frequency on different configurations of the tower.

1. Introduction

The dynamic behavior of structures like ocean tower is an area of extensive research. These structures are widely used to support superstructures like wind turbine, bridges etc. The ocean towers are often subjected to dynamic loads such as wind, waves, etc. Hence, it is important to analyze the dynamic behavior of such structures. These dynamic behaviors can be predicted with reasonable accuracy if the structures are modeled as beam with tip mass and non-classical foundation. Hence, a lot of research has been done to study the problem in this field. For example, Wu and Hsu (2007) analyzed the free vibration of partially wet, elastically supported uniform Euler–Bernoulli beam with eccentric tip mass using two separate sets of analytical formulation. Uscilowska and Kolodziej (1998) provided closed form solution for a partially immersed cantilever beam with eccentric tip mass. Auciello and Ercolano (2004) provided solution for the non-uniform Timoshenko beam to solve the free vibration by the energy method. Wu and Chen (2005) solved the free vibration of non-uniform partially wet Euler–Bernoulli beam with elastic foundation and tip mass. De Rosa et al. (2013) calculated closed form solution for free vibration of a linearly tapered, partially immersed, elastically supported column (Euler–Bernoulli beam) with eccentric tip mass. Wu and Chen (2010) studied the wave-induced vibrations of an axial-loaded, immersed, uniform Timoshenko beam carrying an eccentric tip mass with rotary inertia using analytical formulation.

The vibration analysis of non-uniform Timoshenko beam as ocean tower has been rarely investigated as the authors know. In this work, the ocean tower is modeled as a partially submerged, non-uniform Timoshenko beam supported by a rigid tip mass with eccentricity at the free end, and non-classical damped foundation on the other end. The effect of shear deformation and rotary inertia in included in the beam. The free vibration equation is derived using Hamilton's variational principle based on Rayleigh Ritz Method (RRM) and Finite Element Method (FEM). The solution obtained by these two approaches show a good agreement in results. In RRM, the trial function, which is used to obtain non-uniform beam mode-shapes of ocean tower, is assumed as uniform beam mode-shapes satisfying the boundary conditions of ocean tower. In FEM, the Mindlin-type linear beam element of C^0-order with four degrees of freedom, as explained in Bathe (1996), has been used as shape function. In order to avoid shear locking, reduced integration technique has been incorporated in FEM as explained later. Some of the results are also compared with one present in the existing literature for the verification of computer program and model.

The methodology (Hamilton's variational principle) involves an integral equation and hence, higher order non-uniformity in section area of beam can be handed easily. In both the approaches, i.e., RRM and FEM, the free vibration equation is derived by using this methodology. The difference 4between RRM and FEM lies in choosing the shape function for finding the solution. In RRM, the shape function is chosen over the entire beam while in FEM, it is chosen only for one element. Hence, the main challenge of finding the solution through RRM is satisfying all the boundary conditions using the same shape function. However, the benefit lies in its computational efficiency which is much higher than FEM. This is shown in the convergence study (Section 4.2) where the solution obtained by RRM

* Corresponding author.
E-mail addresses: ankitnaval05@gmail.com (. Ankit), ndatta@naval.iitkgp.ernet.in (N. Datta).

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Nomenclature

x	space variable along the beam length.
t	time variable
u	transverse deflection of non-uniform beam
θ	pure bending slope of non-uniform beam
q	principle coordinate of non-uniform beam
ω	natural frequency of non-uniform beam
ω_R	real part of ω
ω_I	imaginary part of ω
L	length of the beam without tip-mass
D_b	diameter of the base of the tower
D_p	diameter of the upper uniform part of the tower
m_{p0}	reference tip mass
I_{p0}	reference rotary inertia of tip mass
m_p	tip mass
I_p	rotary inertia of tip mass
e_p	eccentricity of tip mass
α	submergence ratio
β	tapering ratio
γ	tip mass ratio
ζ	rotary inertia ratio of tip mass
ρ	density of steel
ρ_w	density of water
M	bending moment in non-uniform beam
V	shear force in non-uniform beam
A	section area of non-uniform beam
I	sectional area 2nd moment of non-uniform beam
C_A	added mass coefficient
k_{t0}	reference translational spring constant
k_{r0}	reference rotational spring constant
k_t	translational spring constant of non-uniform beam
k_r	rotational spring constant of non-uniform beam
κ	Foundation spring stiffness ratio
κ'	foundation spring stiffness ratio as used by Wu and Chen (2010)
c_t	translational damping constant of non-uniform beam
c_r	rotational damping constant of non-uniform beam
η_r	proportionality constant of c_r
η_t	proportionality constant of c_t
E	modulus of elasticity of the material
G	shear modulus
k_0	shape factor of the cross-section
ν	Poisson's ratio
u_F	transverse deflection of uniform beam
θ_F	pure bending slope of uniform beam
q_F	principle coordinate of uniform beam
ω_F	natural frequency of uniform beam
$\omega_{F,R}$	real part of ω_F
$\omega_{F,I}$	imaginary part of ω_F
M_F	bending moment in uniform beam
V_F	shear force in uniform beam
A_F	section area of uniform beam
I_F	sectional area 2nd moment of uniform beam
g	gravitational acceleration
φ	uniform beam mode-shape (trial function)
ϕ	non-uniform modeshape
ψ	uniform pure bending slope mode-shape (trial function)
ψ	non-uniform pure bending slope mode-shape
n	no. of trial functions considered
U	potential energy
T	kinetic energy
R	Rayleigh dissipation factor
M	mass matrix
C	damping matrix
K	stiffness matrix
ξ	local space coordinate
L_e	length of the beam element
U^e	potential energy of beam element
T^e	kinetic energy of beam element
R^e	Rayleigh dissipation factor of beam element
W_g^e	work done due to gravity on beam element
n_e	Nth element of the beam
N_e	total number of beam elements

converges for much lower number of trial functions (e.g., 25) as compared to the number of elements (e.g., 400) in FEM. This makes the matrix size, in eigenvalue problem of RRM, much smaller leading to faster computation.

As we will see later, the number of trial functions needed for convergence of natural frequency is related to the non-uniformity in the beam. If the non-uniformity is more, more trial functions are needed. As mentioned earlier, the trial functions are assumed as uniform beam mode-shapes satisfying the same boundary condition as the ocean tower. Including the effect of rotary inertia and shear deformation in the uniform beam ensures that accurate higher order mode-shapes will be obtained. This is the main motivation behind using a Timoshenko beam over Euler–Bernoulli beam. In case of Euler–Bernoulli beam, the effect of rotary inertia and shear deformation is ignored due to which accurate solutions are limited to lower order mode-shapes only, hence, cannot be used as trial functions in RRM for the analysis of highly non-uniform beams. Beside this, using a Timoshenko beam allows the analysis of thick beams as well.

2. Problem formulation

A continuous, elastic model of an ocean tower is shown in Fig. 1. The tower is modeled as Timoshenko beam. The length of the tower without the tip mass is L. It is immersed up to αL. The foundation of the tower is partially constrained against translation and rotation. Hence, it can be modeled as a combination of translational and rotational linear-springs, with spring constants k_t and k_r, respectively. To account for damping effect due to loose silt at sea bed, translational and rotational linear-dampers, with damping constants c_t and c_r, respectively are

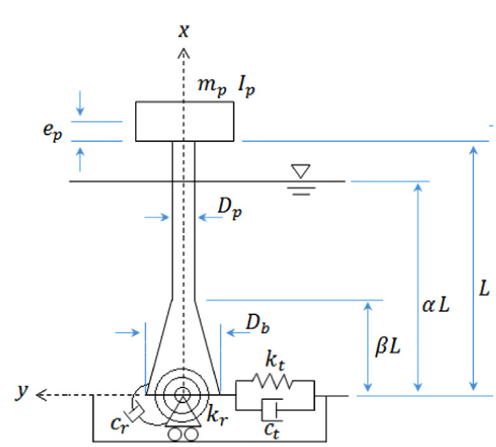

Fig. 1. Ocean tower model.