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# A nonstiff solution for the stochastic neutron point kinetics equations

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ABSTRACT

We propose an approach to solve the stochastic neutron point kinetics equations using an adaptation of the diagonalization-decomposition method (DDM). This new approach (Double-DDM) yields a nonstiff solution for the stochastic formulation, allowing the calculation of the neutron and precursor densities at any time of interest without the need of using progressive time steps. We use Double-DDM to compute results for stochastic problems with constant, linear, and sinusoidal reactivities. We show that these results strongly agree with those obtained by other approaches established in the literature. We also compute and analyze the first four statistical moments of the solutions.



## 1. Introduction

The neutron point kinetics equations (Hetrick, 1971; Kinard and Allen, 2004; Hayes and Allen, 2005) are the coupled differential equations for the neutron density and for the delayed neutron precursor concentrations. These equations model the time-dependent behavior of a nuclear reactor and provide insight into the dynamics of its operation. The time-dependent parameters in this system are the reactivity function and the neutron source term.

The neutron density and delayed neutron precursor concentrations vary randomly with time; however, the point kinetics equations are deterministic and can only be used to estimate average values. Random fluctuations in the neutron density and precursor concentrations can be significant at low power levels (Hurwitz et al., 1963), which points to the importance of estimating these variations.

Hayes and Allen (2005) have generalized the standard deterministic point kinetics equations, deriving a system of stochastic differential equations that model the random behavior of the neutron density and the precursor concentrations in a point reactor. Due to the issue of stiffness, this system was implemented numerically using a stochastic piecewise constant approximation method (Stochastic PCA). Work performed by Saha Ray (2012) has shown that order 1.5 strong Taylor and Euler–Maruyama numerical methods are valid computational alternatives to Stochastic PCA in solving the stochastic point kinetics equations. However, with the exception of cases modeled with either none or only one group of neutron precursors, the stiffness of the problem remains.

In this paper we propose to solve this stochastic formulation using a double decomposition approach based on the diagonalization-decomposition method (DDM) decribed by Wollmann da Silva et al. (2014). This proposed method is the major novelty and principal contribution of this work, yielding a *nonstiff* solution for the stochastic point kinetics equations. Specifically, this approach allows the calculation of the neutron and precursor densities at any time of interest without the need of using progressive time steps. This solution is obtained with a minimal amount of numerical approximations of the model; the largest numerical effort lies in the truncation of the decomposition and the integrations required by DDM.

The major caveat in this approach is that convergence of DDM is yet to be proven. For this reason, a Lyapunov criterion (Boichenko et al., 2005) is used to guarantee convergence (cf. Petersen et al., 2011; Wollmann da Silva et al., 2014). We present computational results for problems with constant, linear, and sinusoidal reactivities. The results of the proposed method are compared against those of other approaches established in the literature, showing strong agreement. We also compute the first four statistical moments of the solutions.

This work is an expanded version of a recent conference paper (Wollmann da Silva et al., 2015). The remainder of this paper is organized as follows. In Section 2 we present a brief review on the key aspects of DDM. In Section 3 we formulate the stochastic

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point kinetics equations. We introduce the proposed double decomposition approach in Section 4. Numerical results are given in Section 5 for problems with constant (Section 5.1) and time-dependent (Section 5.2) reactivities. The paper concludes in Section 6 with a discussion of the work presented.

## 2. The diagonalization-decomposition method (DDM)

Following the work performed by Wollmann da Silva et al. (2014), one can obtain an analytical representation for the solution of the neutron point kinetics equations that is free of stiffness. The neutron point kinetics equations with six groups of precursors and time-dependent reactivity $\rho(t)$ are written as:

$$\frac{d}{dt}n(t) = \frac{\rho(t)-\beta}{\Lambda}n(t) + \sum_{i=1}^{6}\lambda_i C_i(t), \quad n(0) = n_0, \tag{2.1a}$$

$$\frac{d}{dt}C_i(t) = \frac{\beta_i}{\Lambda}n(t) - \lambda_i C_i(t), \quad C_i(0) = \frac{\beta_i n_0}{\Lambda\lambda_i}, \tag{2.1b}$$

for $i = 1, 2, \ldots, 6$. Here, $n(t)$ is the neutron density; $C_i(t)$ is the density of the $i$th delayed neutron precursor group; $\lambda_i$ is the decay constant for a specific group $i$; $\Lambda$ represents the neutron mean generation time; and $\beta_i$ represents the delayed-neutron fraction in a specific group $i$. The total fraction of delayed neutrons is given by $\beta = \sum_{i=1}^{6}\beta_i$.

A recursive scheme with finite recursive depth $R$ is used to obtain a solution. The truncation index is determined with exponential convergence by the Lyapunov criterion (Boichenko et al., 2005; Petersen et al., 2011), evaluated after each recursion step. The neutron population and the precursors concentrations are written in terms of the solution from a recursion initialization ($j = 0$) and the respective correction terms ($j > 0$) for an appropriate $R \in \mathbb{N}$ :

$$n(t) = \sum_{j=0}^{R} n_j(t), \tag{2.2a}$$

$$C_i(t) = \sum_{j=0}^{R} C_{ij}(t). \tag{2.2b}$$

The combination of Eqs. (2.1) and (2.2) yields a system with $7 \times R$ unknowns. We define

$$\boldsymbol{Y}(t) = \sum_{j=1}^{R}\boldsymbol{Y}_j(t), \tag{2.3a}$$

$$\boldsymbol{Y}_j(t) = [n_j(t), C_{1j}(t), C_{2j}(t), C_{3j}(t), C_{4j}(t), C_{5j}(t), C_{6j}(t)]^T, \tag{2.3b}$$

$$\boldsymbol{\Omega} = \mathrm{diag}\left(\frac{\rho_0-\beta}{\Lambda}, -\lambda_1, -\lambda_2, -\lambda_3, -\lambda_4, -\lambda_5, -\lambda_6\right), \tag{2.3c}$$

and

$$\boldsymbol{\Xi} = \begin{bmatrix} \frac{\rho_1(t)}{\Lambda} & \lambda_1 & \lambda_2 & \ldots & \lambda_6 \\ \frac{\beta_1}{\Lambda} & 0 & 0 & \ldots & 0 \\ \frac{\beta_2}{\Lambda} & 0 & 0 & \ldots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \frac{\beta_6}{\Lambda} & 0 & \ldots & 0 & 0 \end{bmatrix}, \tag{2.3d}$$

where the constant $\rho_0$ and $\rho_1(t)$ are such that $\rho(t) = \rho_0 + \rho_1(t)$. Given the recursive system

$$\frac{d}{dt}\boldsymbol{Y}_0(t) - \boldsymbol{\Omega}\boldsymbol{Y}_0(t) = 0, \tag{2.4a}$$

$$\frac{d}{dt}\boldsymbol{Y}_j(t) - \boldsymbol{\Omega}\boldsymbol{Y}_j(t) = \boldsymbol{\Xi}(t)\boldsymbol{Y}_{j-1}(t), \qquad j > 0\,, \tag{2.4b}$$

the solution of Eq. (2.4a) is

$$\boldsymbol{Y}_0(t) = \exp(\boldsymbol{\Omega}t)\boldsymbol{Y}_0(0), \tag{2.5a}$$

with $\boldsymbol{Y}_0(0) = [n_0, C_1(0), C_2(0), \ldots, C_6(0)]^T$. Equation (2.4b) may be formally solved by the Laplace transform:

$$\boldsymbol{Y}_j(t) = \exp(\Omega t)\cancelto{0}{\boldsymbol{Y}_j(0)} + \int_0^t \exp(\boldsymbol{\Omega}\tau)\boldsymbol{\Xi}(t-\tau)\boldsymbol{Y}_{j-1}(t-\tau)d\tau, \qquad j > 0, \tag{2.5b}$$

since the initial condition from Eqs. (2.1) is fully absorbed in Eq. (2.5a). The integral in Eq. (2.5b) is evaluated using the Gauss–Legendre method.

A flowchart describing the implementation of this method is given in Fig. 1. The solution is obtained in an analytical representation that may be evaluated for any time value (free of stiffness).

## 3. The stochastic formulation

Hayes and Allen (2005) derived a system of Itô stochastic differential equations that model the dynamics of the neutron density and the delayed neutron precursors in a nuclear reactor. This formulation describes the variation of the population and can be interpreted as a balance between deaths, births, and transformations of neutrons in the system. The probabilities of these events are determined by the physical parameters of the model, such as the total and partial delayed neutron fractions; the fraction of delayed neutrons of each precursor group; the decay constant of each group; and the average number of neutrons produced in each fission.

Assuming a time interval small enough such that only one event occurs, one can write

$$\frac{d}{dt}\boldsymbol{Y}(t) = \widehat{\boldsymbol{A}}(t)\boldsymbol{Y}(t) + \boldsymbol{Q}(t) + \widehat{\boldsymbol{B}}^{\frac{1}{2}}(t)\frac{d}{dt}\boldsymbol{W}(t), \tag{3.1a}$$

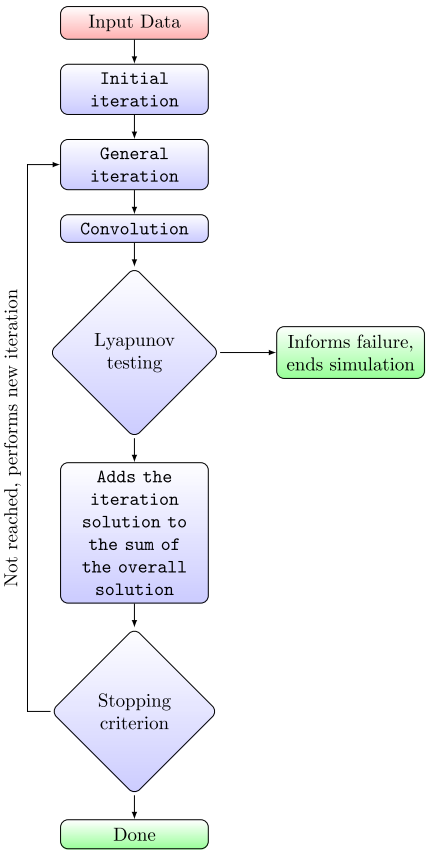


**Fig. 1.** DDM approach to solve the deterministic problem.