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A mechanistic semi-empirical wake interaction model for wind farm layout optimization

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ABSTRACT

Optimizing the turbine layout in a wind farm is crucial to minimize wake interactions between turbines, which can lead to a significant reduction in power generation. This work is motivated by the need to develop wake interaction models that can accurately capture the wake losses in an array of wind turbines, while remaining computationally tractable for layout optimization studies. Among existing wake interaction models, the SS (sum of squares) model has been reported to be the most accurate. However, the SS model is unsuitable for wind farm layout optimization using mathematical programming methods, as it leads to non-linear objective functions. Hence, previous work has relied on approximated power calculations for optimization studies. In this work, we propose a mechanistic linear model for wake interactions based on energy balance, with coefficients determined based on publicly available data from the Horns Rev wind farm. A series of numerical experiments was conducted to assess the performance of the wake interaction model. Results show that the proposed model is compatible with standard mathematical programming methods, and resulted in turbine layouts with higher energy production than those found using previous work.

1. Introduction

Wind energy is one of the fastest growing sustainable sources of electricity, experiencing exponential growth in recent years [1]. Wind turbines extract kinetic energy from the wind through interactions between the turbine blades and the wind. The aerodynamic forces produced by the wind turns the rotor to generate electricity. The air behind the turbine is slowed down with its turbulence intensity increased [2]. This region of decelerated air is called the wake [2]. The wake expands as it travels downstream, mixing with surrounding air and increasing its velocity back to undisturbed conditions after some distance. This distance is crucial as the performance of turbines downstream is dependent on the incoming wind conditions. If turbines are too close to each other, the wind cannot recover to its upstream state [3], causing losses in energy generation for turbines downstream [4–6]. These wake losses can reduce the annual energy production by as much as 10%–20% [4]. Therefore, an understanding of turbine wakes is crucial for optimal wind farm planning [3].

Determining the optimal layout of a set of wind turbines in a given area is a complex problem. Wind farm layout problems can be modelled in two ways, namely (1) continuous and (2) discrete. In discrete models, the turbines can only be placed in a countable set of pre-determined locations inside the wind farm, while in continuous models they can be placed anywhere in the farm, considering their coordinates as continuous variables. Metaheuristic algorithms such as evolutionary algorithms [7–11], particle swam optimization [12,13], and extended pattern search [14], have been the primary tools to solve continuous models. Although powerful in tackling non-linear problems, metaheuristics cannot guarantee global optimality.

There are a number of publications on discrete modelling of wind turbine placement in wind farms [15–18]. An example is the work by Mosetti et al. [15], where the wind farm is divided into 10 by 10 square cells and each turbine is placed in the centre of each cell. The cell sizes are chosen to enforce distance constraints between turbines, e.g., turbines cannot be placed closer than five turbine diameters apart. Although layout solutions to

Abbreviations: LSVD, linear superposition of velocity deficits; LS, linear superposition; SED, sum of energy deficits; SS, sum of squares; SKED, sum of kinetic energy deficits; GS, geometric superposition.

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discrete models may be of lower spatial resolution than of continuous models, discrete models can be solved using powerful mathematical programming approaches [19–21], which can guarantee optimality of the solutions for linear and quadratic functions and constraints. In a well-designed discrete model, knowing the optimality of solutions can save tremendous amount of time in the optimization process.

A MIP (mixed integer programming) problem consists of an objective function and a mix of integer and continuous constraints. The layout optimization problem can be modelled in this mathematical programming approach by discretizing the wind farm domain into possible turbine placement locations, with binary decision variables denoting if a turbine is placed at a specific location or not. These problems can be solved using algorithms such as branch and bound [22]. Fagerfjäll [23], Donovan et al. [19], Zhang et al. [20], and Turner et al. [21] have studied the application of branch and bound methods in wind farm layout optimization. Applying mathematical programming methods to solve the layout problem is promising due to the optimality of the solutions can be known, as opposed to meta-heuristics that provide no guarantee of convergence. Furthermore, solver efficiency can be improved through alternative problem formulations and problem-specific branching strategies [20]. The objective of this paper is to introduce a novel wake interaction model that leads to linear MIP formulations, thus guaranteeing the optimality of solutions to WFLO (wind farm layout optimization) problems.

2. Wake modelling

2.1. Single wake model

The Jensen model [24] is one of the most widely used wake models. It assumes a linearly expanding wake and uniform velocity profile inside the wake. As a result of momentum conservation, the decelerated wind behind the rotor recovers to free stream speed after travelling a certain distance downstream of the turbine [24]. The velocity downstream from the rotor is given by

$$u(x) = U_\infty \left[1 - \frac{1-\sqrt{1-C_T}}{\left(1+2k\frac{x}{D}\right)^2} \right], \tag{1}$$

where C_T is the thrust coefficient of the turbine, D is the turbine rotor diameter, U_∞ is the wind speed in the free stream, and k is the Wake Decay Constant, which is generally taken to be 0.075 for onshore farms and 0.04 to 0.05 for offshore farms.

The power production of each turbine i is based on the incoming wind speed that it experiences,

$$P_i = \frac{1}{2}\rho A u_i^3 (\eta_{gen} C_P), \tag{2}$$

where A as the rotor area, ρ is the density of the air, η_{gen} is the generator efficiency, and C_P is the rotor power coefficient. The AEP (annual energy production) of a wind farm is defined as the integration of power production (kW) over time (hr),

$$AEP = 8766 \sum_{i=1}^{N} \sum_{d \in L} p_d P_{i,d}. \tag{3}$$

where p_d is the probability of wind state d, defined as a (speed, direction) pair, L is the set of wind states with non-zero probability for the specific wind farm site, N is the total number of turbines, and 8766 is the effective number of hours in a year.

2.2. Wake interaction models

The interaction of multiple superimposed turbine wakes is not fully understood, as it involves complex turbulence phenomena [25]. A number of descriptions exist in the literature to determine the wind speed due to the presence of multiple turbine wakes upstream. In particular, four descriptions available in the literature [2], listed in Table 1, will be introduced in this section. In these equations, u_i is the wind speed at turbine i, u_{ij} is the wind speed at turbine i due to (the wake of) turbine j and the summations and the products are taken over the n turbines upstream of turbine i [2,26,27].

The GS (Geometric Superposition) assumes the ratio of the wind speed at a location relative to the free stream speed is a product of velocity ratios caused by upstream turbines. The LSVD (linear superposition of velocity deficits) considers that the velocity deficit at a given turbine is equal to the sum of the velocity deficits caused by all turbines upstream from it. The SED (Sum of Energy Deficits) assumes the kinetic energy deficit in the wakes is additive. The SS (Sum of Squares) sums up the squares of the velocity deficits of the upstream wakes.

Two main issues exist that hinder the use of these wake interaction models. Firstly, with the exception of SED, the physical basis of these descriptions is unclear, which makes improvement through experimental data difficult. Secondly, using all of the mentioned wake interaction models in deterministic optimization methods remains a challenge, for several reasons. If the objective function of the WFLO problem is to maximize power or energy production, all of the above wake interaction models (except SED) would lead to non-linear objective functions and linear approximations will be required to improve solvability. In addition, all of the models mentioned are recursive functions. Specifically, the term u_j, i.e. the incoming wind speed that a turbine j experiences, is dependent on the conditions upstream, which are unknown a priori, thus precluding the use of well-established mathematical programming methods for its solution [19–21].

In the literature, comparisons of the different wake interaction models with experimental measurements have demonstrated that the sum of squares model, despite lacking physical meaning, is the most accurate [2]. However, as mentioned previously, it is difficult to implement sum of squares into a MIP formulation. In this work, a wake interaction model based on the principle of energy balance is presented as a physics-based, linear alternative to the sum of squares model, leading to linear objective functions.

3. Proposed wake model

3.1. Energy balance

In the proposed wake interaction model, energy balance is used to describe the wind speed recovery in the far wake, where the flow is fully developed [3]. The speed recovery in the wake is due to

Table 1
Wake interaction models.

Name	Formula
Geometric superposition (GS)	$\frac{u_i}{U_\infty} = \prod_{j=1}^{n} \frac{u_{ij}}{u_j}$
Linear superposition of velocity deficits (LSVD)	$\left(1-\frac{u_i}{U_\infty}\right) = \sum_{j=1}^{n}\left(1-\frac{u_{ij}}{u_j}\right)$
Sum of energy deficits (SED)	$(U_\infty^2 - u_i^2) = \sum_{j=1}^{n}(u_j^2 - u_{ij}^2)$
Sum of squares (SS)	$\left(1-\frac{u_i}{u_\infty}\right)^2 = \sum_{j=1}^{n}\left(1-\frac{u_{ij}}{u_j}\right)^2$