Journal of Petroleum Science and Engineering 140 (2016) 149–153

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

Calculation of pressure distribution in heavy oil reservoir with boundary element method

Xuehui Wang [a], Guiyang Ma [b,*], Yujie Dai [a]

[a] College of Sciences, Liaoning Shihua University, Fushun 113001, China
[b] College of Petroleum Engineering, Liaoning Shihua University, No.1, West Dandong Road, Wanghua District, Fushun, Liaoning Province 113001, China

ARTICLE INFO

Article history:
Received 1 January 2015
Received in revised form
15 January 2016
Accepted 18 January 2016
Available online 19 January 2016

Keywords:
Boundary element method
Seepage
Pressure distribution
Numerical simulation
Temperature

ABSTRACT

Boundary element method (BEM) is introduced to calculate the heavy oil seepage in this paper. The pressure contours with different temperatures and arbitrary oil reservoir shapes are obtained by solving the variable coefficients Darcy's equations. The numerical simulation results indicate that temperature has evident effect on pressure contours when the pressure is lower. Varieties of pressure contours with different boundary conditions coincide well with the heavy oil seepage law, which indicates the validity and accuracy of BEM in dealing with the irregular boundary problem. The numerical simulation results can provide theoretical basis for the heavy oil well pattern.

1. Introduction

Seepage law of porous media is a highly concerned problem in the engineering. Many ways are used to study the evolution of seepage up to the present. Georgette et al. (2014) studied the porous media flows with heterogeneous modeling method. Numerical simulation results show that the hierarchically coupled models accurately account for the heterogeneity of the medium and efficiently incorporate local features into the global response. New variational inequality formulation is presented by Zheng et al. (2005) and seepage problems with free surfaces is studied.

In fact, most available research methods for porous media flow are based on solving the Darcy's equations (Hou et al., 2013; Dong et al., 2013; Wang et al., 2010; Lv et al., 2014; Rafiezadeh and Ashtiani, 2014; Liu et al., 2014). For all seepage problems, low permeability seepage law in heavy oil reservoirs is one of the most important subjects. For a long time, geometrical shapes are considered regular or completely closed and boundary pressure is constant when we measure the heavy oil pressure. Actually, the boundary pressures and shapes are various which indicate the simplified model mentioned above is unreasonable (Zhang et al., 2006). Because of the complexity of boundary in arbitrary shapes and different pressures, it is extremely difficult to solve the problems with conventional computational methods.

BEM is a new numerical method after the finite difference method and the finite element method and widely used in treating complex boundary problems for its flexibility. There are two steps when we solve a definite solution problem with BEM. The first is change the definite solution problem into a Green function problem with specific boundary condition. This step can reduce the dimensions of the given problem and increase computational efficiency. The second is the discretization of boundary. Values on boundary are discrete and in internal points are analytic, so the computational precision of BEM is higher than that of other methods.

Considering the flexibility of BEM in handle the boundary conditions (Kikani and Horne, 1988; Kikani and Horne, 1989), we will study the evolution of heavy oil seepage with this method in this paper.

2. Numerical method and basic equations

2.1. Mathematical model and boundary integral equation

Engineering seepage problem come down to solve Darcy's equations, general vector form of Darcy's equations can be written as

$$\boldsymbol{V} = -\boldsymbol{K}\cdot\nabla u \tag{1}$$

Scalar formulas of Eq. (1) are the following:

* Corresponding author.
E-mail address: gyma2014@126.com (G. Ma).

http://dx.doi.org/10.1016/j.petrol.2016.01.022
0920-4105/

$$\begin{cases} v_x = -k_x \dfrac{\partial u}{\partial x} \\ v_y = -k_y \dfrac{\partial u}{\partial y} \\ v_z = -k_z \dfrac{\partial u}{\partial z} \end{cases} \tag{2}$$

where v_x, v_y and v_z are the Darcy's velocities, k_x, k_y and k_z are the permeability coefficients in x, y and z directions separately. In addition, k_x, k_y and k_z are not constants but functions of temperature, water content, asphalt content and porosity et al. u is potential function and equivalent to the pressure p.

Suppose $Q(x,y,z)$ is the source in the internal domain Ω, then the continuity equation can be written as

$$\nabla \cdot \boldsymbol{V} = -Q \tag{3}$$

Governing equation can be obtained by combining Eqs. (2) and (3).

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial u}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial u}{\partial z}\right) = Q \tag{4}$$

We consider two dimension (2-D) problems in this paper, and then Eq. (4) can be written as

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial u}{\partial y}\right) = Q \tag{5}$$

Then the 2-D seepage problem can be change into the following definite solution problem:

$$\begin{cases} \dfrac{\partial}{\partial x}\left(k_x \dfrac{\partial u}{\partial x}\right) + \dfrac{\partial}{\partial y}\left(k_y \dfrac{\partial u}{\partial y}\right) = Q & \text{in} \quad \Omega \\ q = k_x \dfrac{\partial u}{\partial x} n_x + k_y \dfrac{\partial u}{\partial y} n_y = \bar{q} & \text{on} \quad \Gamma_1 \\ u = \bar{u} & \text{on} \quad \Gamma_2 \end{cases} \tag{6}$$

where Γ denotes the whole boundary of heavy oil reservoirs, Γ_1 and Γ_2 denote partial boundary and satisfied $\Gamma = \Gamma_1 + \Gamma_2$. q is normal derivative. The boundary can be divided into n parts and satisfied $\Gamma = \Gamma_1 + \Gamma_2 + \Gamma_3 + \cdots + \Gamma_n$ if the boundary conditions are more complex, $\bar{q}$ and $\bar{u}$ on each part are known qualities.

For solve definite solution problem of Eq. (6), weighted residual method is used to get the boundary integral equation. The weighted residual formula is shown in Eq. (7)

$$\int_\Omega \left[\frac{\partial}{\partial x}\left(k_x \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial u}{\partial y}\right)\right] u^* \, d\Omega - \int_\Omega Q u^* \, d\Omega = 0 \tag{7}$$

Eq. (8) will be obtained by calculated Eq. (7) with integral method.

$$-u_i + \int_\Gamma u^* q \, d\Gamma - \int_\Gamma q^* u \, d\Gamma + \int_\Omega \left(\frac{\partial k_x}{\partial x}\frac{\partial u^*}{\partial x} + \frac{\partial k_y}{\partial y}\frac{\partial u^*}{\partial y}\right) u \, d\Omega = \int_\Omega u^* Q \, d\Omega \tag{8}$$

The boundary integral equation is given in Eq. (9). Details of the computational process can be obtained in Yang and Zhao (2002).

$$\frac{1}{2}u_i - \int_\Gamma u^* q \, d\Gamma + \int_\Gamma q^* u \, d\Gamma - \int_\Omega \left(\frac{\partial k_x}{\partial x}\frac{\partial u^*}{\partial x} + \frac{\partial k_y}{\partial y}\frac{\partial u^*}{\partial y}\right) u \, d\Omega = -\int_\Omega u^* Q \, d\Omega \tag{9}$$

where $u_i = u(x_i, y_i)$, q^* and u^* are directional derivative and fundamental solution, respectively.

$$q^* = k_x \frac{\partial u^*}{\partial x} n_x + k_y \frac{\partial u^*}{\partial y} n_y \tag{10}$$

$$u^* = \frac{1}{2\pi\sqrt{k_x k_y}} \ln\left(\frac{1}{r_c}\right) \tag{11}$$

where u^* satisfies the relationship of Eq. (12) and formula of r_c is given in Eq. (13)

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial u^*}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial u^*}{\partial y}\right) + \delta(x, y, x_i, y_i) = 0 \tag{12}$$

$$r_c = \sqrt{[(x - x_i)^2/k_x] + [(y - y_i)^2/k_y]} \tag{13}$$

$\delta(x, y, x_i, y_i)$ is *dirac* $-$ δ function, (x, y) are observation points and (x_i, y_i) are point source. To calculate the unknown quantities u and q on the boundary, boundary integral equation must be calculated. However, it is impossible to find the analytical solution of Eq. (9) for certain boundary condition. Therefore, we resort to numerical means.

2.2. Discretization of boundary integral equation

The discretization forms are different from equation to equation (Lashgari, 2014). The discretization forms in this paper are as follows. The internal domain Ω is divided into m units Ω_1, Ω_2,…, Ω_m. The whole boundary will be divided into $(n-m)$ units Γ_{m+1}, Γ_{m+2},…, Γ_n. Based on the characteristic of constant value unit, value of each unit can be replaced by the node point value u_j and q_j. Then the discretization form of Eq. (9) can be written as

$$\frac{1}{2}u_i - \sum_{j=m+1}^{n} \int_{\Gamma_j} (u^* d\Gamma) q_j + \sum_{j=m+1}^{n} \int_{\Gamma_j} (q^* d\Gamma) u_j = -\sum_{j=1}^{m} \int_{\Omega_j} u^* Q \, d\Omega \tag{14}$$

For convenience, we let

$$\hat{H}_{ij} = \int_{\Gamma_j} q^* d\Gamma \tag{15}$$

$$G_{ij} = \int_{\Gamma_j} u^* d\Gamma \tag{16}$$

$$B_i = \sum_{j=1}^{m} \int_{\Omega_j} u^* Q \, d\Omega \tag{17}$$

Then Eq. (14) change into

$$\frac{1}{2}u_i + \sum_{j=m+1}^{n} \hat{H}_{ij} u_j = \sum_{j=m+1}^{n} G_{ij} q_j - B_i \tag{18}$$

Let u^i is the value of each unit midpoint, $u^i = u_i$ when $i = j$, we can let

$$H_{ij} = \hat{H}_{ij} + \frac{1}{2}\delta_{ij} \tag{19}$$

Then the algebraic equations can be written as

$$\sum_{j=m+1}^{n} H_{ij} u_j = \sum_{j=m+1}^{n} G_{ij} q_j - B_i \tag{20}$$

Correlation discretization forms of coefficient matrices are given when $i = j$ by combining Eqs. (10) and (11) and Eqs. (15) and (16).