Journal of Petroleum Science and Engineering 134 (2015) 214–222

Contents lists available at ScienceDirect

Journal of Petroleum Science and Engineering

journal homepage: www.elsevier.com/locate/petrol

Risk-controlled wellbore stability analysis in anisotropic formations

Majed F. Kanfar*, Z. Chen, S.S. Rahman

School of Petroleum Engineering, University of New South Wales, Australia

ARTICLE INFO

Article history:
Received 12 September 2014
Received in revised form
2 June 2015
Accepted 6 August 2015
Available online 7 August 2015

Keywords:
Elastic anisotropy
Strength anisotropy
Wellbore stability
Shear failure
Tensile failure
Controlled stability

ABSTRACT

The paper presents a semi-analytical solution for the stability of inclined boreholes drilled in isotropic and anisotropic formations. Conventional analytical or semi-analytical models are designed to yield no borehole collapse failure along the borehole wall. However, in deep wells, where there are high hoop stresses acting on the borehole wall, it is difficult to apply the safe mud weight produced by these ubiquitous models. The proposed solution in this work imposes a constraint on how much failure will occur along the borehole wall. This risk-controlled stability analysis will produce a safe mud window that can be realistically achieved during drilling operations. Analytical solutions for stress distribution for isotropic and anisotropic rocks are presented. In addition, a solution for the upper limit for the mud window to prevent tensile failure is developed. The initial poroelastic or "undrained" drilling effect on pore pressure is also incorporated in the discussed models.

1. Introduction

Kirsch's solution assumes a linearly elastic isotropic homogeneous material for a circular hole in an infinite plane (Fjaer, 2008; Kirsch, 1898). His equations estimate the drilling-induced or perturbation stresses caused by the creation of a circular cavity in an infinite plane. Assuming isotropy ignores the effect of endogenic processes like diagenesis, and exogenic processes such as compaction and erosion that affect most rocks during its formation. These processes alter the internal rock fabric and consequently its mechanical behavior. It is believed that only 10% of subsurface formations exhibit a true isotropic behavior. More than 30% of rocks classified as anisotropic have an anisotropy ratio (the ratio of the horizontal Young's modulus to the vertical one with respect to the bedding plane) of 1.5 for Young's modulus (Ong, 1994). Due to its simplicity, most formulations for borehole stability models assume an isotropic linear elasticity model. These models have fallen short to simulate an accurate representation of the complex rock anisotropic mechanical frame. The first comprehensive anisotropic linear elasticity analysis was developed by Lekhnitskii (1981). The solution uses the generalized plane strain assumption to account for the in-plane, uniaxial and anti-plane stresses. Amadei (1983) extended Lekhnitskii's work to infinite cylindrical holes. The solution can be used to estimate stresses in anisotropic material angularly around the borehole, and radially away from the borehole wall. It also accounts for the stress field rotation, borehole inclination and formation dipping effect on stress perturbation caused by the hole.

Aadnoy and Chenevert (1987) developed an analytical solution for highly inclined boreholes that assumes an isotropic homogenous rock. Their work was employed by McLean and Addis (1990) to contrast the effect of different shear failure criteria such as Mohr–Coulomb (MC) and Drucker–Prager (DP). They have also incorporated a simplified form of the "undrained" pore pressure effect into the model. However, these criteria ignore the effect of the intermediate principal stress. Several published criteria incorporates all three principal stresses such as the Lade criterion (Lade, 1977) that was later modified to accommodate rocks with cohesive strength (Ewy, 1999). Similar work was developed by Al-Ajmi and Zimmerman (2005, 2006) who extended the Coulomb criterion. They have utilized a linear "Mogi–Coulomb" criterion that accounts for the intermediate principal stress. The Mogi–Coulomb model was contrasted against field data and has demonstrated more realistic results when compared to those produced by MC and Hoek–Brown criteria (Gholami et al., 2014).

The aforementioned authors assume elastic isotropy, and very few authors in literature have used an anisotropic rock model in wellbore stability analyses. Aadnoy (1988) was the first to use Lekhnitskii's seminal work for wellbore stability analysis. Aadnoy specialized the model for inclined boreholes drilled in anisotropic formations using a semi-analytical solution. An exact solution for anisotropic wellbore stability was developed by Ong and Roegiers (1993) using Amadei's work (1983).

In addition to the elastic anisotropy, the rock's anisotropy in strength parameters such as cohesive strength and angle of internal friction need to be considered. Both matrix and properties

* Corresponding author.
E-mail address: m.kanfar@student.unsw.edu.au (M.F. Kanfar).

http://dx.doi.org/10.1016/j.petrol.2015.08.004
0920-4105/

along bedding planes should be incorporated into the failure analysis. Transversely isotropic (TI) rocks tend to slip and slide along their planes of weakness. This is often overlooked and only matrix failure is considered. Several researchers have investigated the effect of rock strength anisotropy on rock shear failure (Aoki et al., 1993; Jaeger et al., 2009; Li et al., 2012; Tien and Kuo, 2001). These models were incorporated into borehole stability analyses, but were coupled with an isotropic elastic rock model (Lee et al., 2012; Zhang, 2013).

In this work, a comprehensive model that couples elastic and strength anisotropy and pore pressure undrained effect is developed to fully depict the rock complex nature. In addition, a novel risk-controlled stability analysis is developed using a semi-analytical approach. The developed model is for cylindrical wellbores with circular cross-sectional areas.

2. Linear elasticity models

Modeling 3D stress problems in anisotropic formations requires establishing three coordinate systems: 1. The far-field stresses coordinate system which is referenced to the North-East-Vertical (NEV) system, 2. The intrinsic rock properties coordinate system and 3. The borehole coordinate system. In oil and gas applications, it is useful to reference all properties to the borehole coordinate system (Aadnoy and Chenevert, 1987). The coordinate systems are summarized in Fig. 1. In the following subsections, the anisotropic and isotropic linear elasticity models are briefly presented and contrasted with an example.

2.1. Anisotropic rock

The generalized Hooke's law in strain–stress form in Voigt notation is given by

$$\epsilon_i = C_{ij}\,\sigma_j\,; \quad i,\ j = 1,\ 2\ \ldots 6 \tag{1}$$

where ϵ, C and σ are the strain, compliance and stress tensors respectively. The compliance tensor for orthorhombic (orthotropic) material in the intrinsic rock properties coordinate system is given by

$$C = \begin{bmatrix} 1/E_x & -v_{yx}/E_y & -v_{zx}/E_z & 0 & 0 & 0 \\ -v_{yx}/E_y & 1/E_y & -v_{zy}/E_z & 0 & 0 & 0 \\ -v_{zx}/E_z & -v_{zy}/E_z & 1/E_z & 0 & 0 & 0 \\ 0 & 0 & 0 & 1/G_{yz} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/G_{xz} & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/G_{xy} \end{bmatrix} \tag{2}$$

where E_i, v_{ij} and G_{ij} ($i,\ j = x, y$ and z are the principal directions of the orthorhombic material) are the Young's modulus, Poisson's ratio and shear modulus, respectively, in the i and j directions. The compliance tensor is rotated to the borehole coordinate system to produce the matrix $\boldsymbol{a}$, where the off diagonal terms may not have zero values. Similarly, the far-field stresses are rotated from the NEV coordinate system to the borehole coordinate system to produce the following matrix:

$$[\sigma_{x,o}\quad \sigma_{y,o}\quad \sigma_{z,o}\quad \tau_{yz,o}\quad \tau_{xz,o}\quad \tau_{xy,o}]^T \tag{3}$$

where $\sigma_{i,o}$ and $\tau_{ij,o}$ ($i,\ j = x, y$ and z are the principal directions of the borehole coordinate system) are the far-field in-situ normal and shear stresses, respectively, rotated to the borehole Cartesian coordinate system. The superscript T refers to the transpose of the matrix. A detailed procedure on the stress and compliance tensor rotation is discussed in (Amadei, 1983).

Lekhnitskii (1981) developed a general analytical solution for anisotropic linear elasticity and was modified by Amadei (1983) for cylindrical holes with infinite boundary. The total stress distribution along the borehole wall in Cartesian coordinates of the borehole coordinate system for anisotropic formations is given by (Amadei, 1983)

$$\sigma_{xx} = \sigma_{x,o} + \sigma_{x,h} = \sigma_{x,o} + 2\Re[\mu_1^2\phi_1'(z_1) + \mu_2^2\phi_2'(z_2) + \lambda_3\mu_3^2\phi_3'(z_3)] \tag{4}$$

$$\sigma_{yy} = \sigma_{y,o} + \sigma_{y,h} = \sigma_{y,o} + 2\Re[\phi_1'(z_1) + \phi_2'(z_2) + \lambda_3\phi_3'(z_3)] \tag{5}$$

$$\tau_{xy} = \tau_{xy,o} + \tau_{xy,h} = \tau_{xy,o} - 2\Re[\mu_1\phi_1'(z_1) + \mu_2\phi_2'(z_2) + \lambda_3\mu_3\phi_3'(z_3)] \tag{6}$$

$$\tau_{xz} = \tau_{xz,o} + \tau_{xz,h} = \tau_{xz,o} + 2\Re[\lambda_1\mu_1\phi_1'(z_1) + \lambda_2\mu_2\phi_2'(z_2) + \mu_3\phi_3'(z_3)] \tag{7}$$

$$\tau_{yz} = \tau_{yz,o} + \tau_{yz,h} = \tau_{yz,o} - 2\Re[\lambda_1\phi_1'(z_1) + \lambda_2\phi_2'(z_2) + \phi_3'(z_3)] \tag{8}$$

$$\begin{aligned}\sigma_z &= \sigma_{z,o} + \sigma_{z,h} \\ &= \sigma_{z,o} - (1/a_{33})[a_{31}\sigma_{x,h} + a_{32}\sigma_{y,h} + a_{34}\tau_{yz,h} + a_{35}\tau_{xz,h} + a_{36}\tau_{xy,h}]\end{aligned} \tag{9}$$

where $\Re$ refers to the real part of the complex expression, $\sigma_{i,h}$ and $\tau_{ij,h}$ ($i,\ j = x, y$ and z are the principal directions of the borehole coordinate system) are the induced stresses created by the excavation of the hole, a_{ij} ($i, j = 1, 2 \ldots 6$) is the compliance tensor rotated to the borehole coordinate system. The analytic functions, ϕ_k', and the other variables are discussed in the Appendix.

2.2. Isotropic rock

The reduced form of Kirsch's solution for total stress distribution along the borehole wall is governed by the following equations in cylindrical coordinates in the r–θ–z directions (Jaeger et al., 2009):

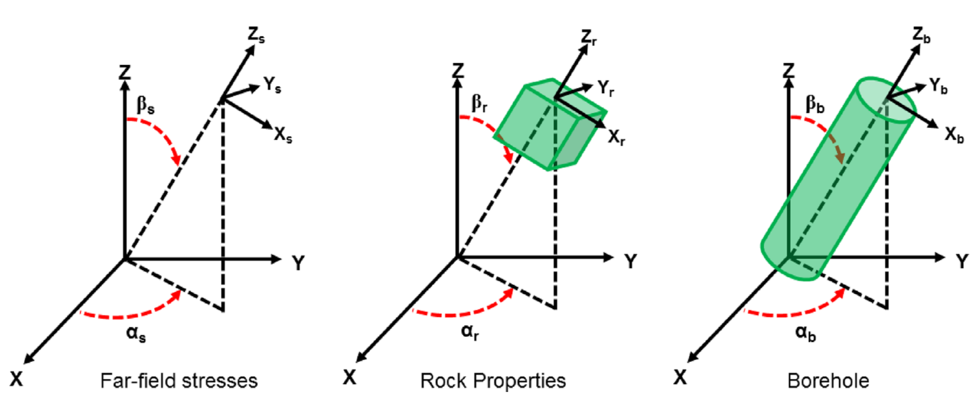

Fig. 1. Coordinate transformation systems.