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A fractal analysis of fracture conductivity considering the effects of closure stress

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ABSTRACT

Hydraulic fracturing and acid fracturing are very important in oil and gas reservoir development. The role of fluid flow in artificial fractures is highly complex, and it has been shown that closure stress plays an important role in fracture conductivity. However, it is difficult to predict closure stress accurately due to the randomness of the acid-rock reaction and various influencing factors. In this work, using digital image processing method, novel predictive models for fracture conductivity that consider closure stress are developed based on the fractal theory and the mechanics of materials. Simulation results show that the Young's modulus E, Poisson's ratio ν, crack spacing fractal dimensions and tortuosity fractal dimensions have significant effects on fracture conductivity. The predictions of the fracture conductivity show good agreement with the available experimental data and indicate that the proposed models can accurately characterize the flow in acid-etched fractures under stress.

1. Introduction

To achieve economical production, most of the wells treated with hydraulic fracturing need to be operational. Fracture conductivity directly influences the productivity of a fractured formation, and it is an important index for judging the success of a fracturing treatment. Gradual reductions in formation pressure and fracture conductivity as well as a gradual increase in closure stress occur as development progresses.

Over the past four decades, many investigators have studied the seepage characteristics of fractures and proposed several fracture conductivity models. Existing acid-etched fracture conductivity models can be divided into three categories: empirical models, theoretical models and digital calculation methods. Empirical models have played a significant role in the study of fracture conductivity. The most widely used model is the N–K model, but its application scope is very limited, which has been confirmed by previous research (Mirza et al., 1996; Nino Penaloza, 2013). Based on the empirical models, many formulas were derived, and Neuzil and Tracy, 1981 modified the probability density function of the fracture aperture, which exhibits the scale effect (Tsang and Witherspoon, 1983; Elsworth and Goodman, 1986). Nitao and Buscheck (1991) modified the contact ratio of the fracture face and derived the fracture conductivity prediction model by considering closure stress for fluid laminar flow in a rough fracture. Dreuzy et al. (2012) proposed a numerical analysis of the combined effect of fracture-scale heterogeneities and the network-scale topology. The upscaling from the fracture to the network scale modified the impact of fracture roughness on the measured permeability. The flow model in individual fractures defined the local flux q field as the integral along the fracture aperture of the flow velocity field, which is identical to the well-known cubic law relating the volumetric flow through a parallel plate fracture to the macroscopic gradient defined at the fracture scale.

A vast amount of research work in acid-etched fracture conductivity at closure stress had been done during the past several years. Mou et al. (2011) developed new correlations of acid fracture conductivity at low closure stress. Assuming elastic behavior of the rock, Deng et al. (2010, 2012) performed many numerical experiments and developed a new acid-fracture conductivity correlation subject to closure stress, in which the deformation of the fracture surfaces was approximated by elliptical model (Jaeger et al., 2007), the study revealed the effects of both permeability and mineralogy distributions, and rock elastic properties on the overall conductivity of an acid etched fracture.

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Ebrahimi AN (2014) developed a model to study the combined effect of fracture scale heterogeneity and network topology on the equivalent permeability of a fractured medium. They assumed that the width between two plates/walls of a fracture, i.e. the parallel plate model is used to represent the effective aperture of a fracture. But all fracture planes are uneven, especially the acid fracturing of carbonate reservoirs. Fracture conductivity depends on the fracture morphology under closure stress. With improvements in the experimental equipment, the most widely used mathematical model representing fracture morphology and fracture conductivity is the roughness coefficient (Pelević and van der Meer, 2016; Zhang et al., 2014; Li and Huang, 2015a; 2015b). The relationship between the crack width and the joint roughness coefficient (JRC) was presented based on an analysis of sufficient experimental data (Barton et al., 1985; Zhao, 1997a; Tatone and Grasselli, 2010; Du et al., 2011; Zoorabadi et al., 2015). After this, many parameters were proposed, such as the mesh coefficient (Zhao, 1997b; Johansson and Stille, 2014) and the asperity height (Ciavarella, 2015; Serpieri et al., 2015; Liou and Lin, 2010; Amiri Hossaini et al., 2014). Considering the uncertainty in describing the geometric characteristics of a rough surface with a unidirectional contour, the ratio of contour line length to projection length in two orthogonal directions was provided to classify the etching patterns of an acid-fracture surface and to calculate the fracture conductivity (Bai and Lai, 2014).

The introduction of fractal theory (Mandelbrot, 1967; 1983) provided a new way of thinking about and describing surface topography and joint closure behavior. Turk et al. (1987) established the Relational Model of Standard JRC and Fractal Dimension. Miao et al. (2015) derived a fractal model for the permeability of rock with random fractures. However, the model did not analyze the effects of closure stress, and the width of the fracture was assumed to be constant.

The main objective of this paper is to develop a theoretical model for acid-etched fracture conductivity under closure stress based on fractal theory and rock mechanics. The rest of this paper is organized into the following sections, the experiment for acid-etched fracture conductivity and fracture plane digitization will be introduced in Section 2, followed by fractal model derivation in Section 3, the impacts of mechanical parameters, fractal dimensions and closure stress on acid-etched fracture conductivity are discussed in Section 4. In the end, some key concluding remarks are presented in Sections 5.

2. Experimental setup

2.1. Experimental procedure

The experimental procedure follows four consecutive steps:

1) Measurement of rock property before acidizing, 2) acid injection, 3) Digitization of the acid-etched fracture, 4) conductivity measurements. All of the experiments presented in this paper were conducted with a polymer gelled acid system (concentration is 20 percent) at a temperature of 110 °C, the injection rate was maintained at 150 ml/min and contact time was 10 min.We have conducted a total of ten experiments.

2.2. Sample preparation

The cores used in this study were taken from the Ordovician carbonate reservoir in the Shun Nan area of central Tarim Basin, which has a deep burial depth. The petrophysical and rock mechanical parameters are shown in Table 1. The Young's moduli (E) and Poisson's ratios (υ) were obtained from uniaxial stress experiments. Then the cores were directly split into halves with the Brazilian technique to obtain fracture samples (Fig. 1), a total of 10 fracture samples were prepared.

Table 1
Petrophysical and rock mechanical parameters of the cores.

Number	L (cm)	E (GPa)	υ/	φ (%)	K (10^{-3} μm^2)
1	4.95	0.13	0.25	5.41	0.64
2	5.09	0.28	0.23	6.09	1.75
3	5.10	0.46	0.18	5.60	0.28
4	4.58	0.15	0.28	5.52	0.21
5	4.84	0.21	0.21	9.73	0.24
6	4.45	0.24	0.26	7.11	0.23
7	5.03	0.18	0.31	2.98	1.48
8	5.00	0.33	0.27	4.97	0.24
9	5.00	0.24	0.23	5.84	0.10
10	5.07	0.16	0.29	2.63	1.96

Many parameters, such as acid-injection condition, acid-diffusion coefficient, acid concentration, and temperature, have an impact on the treatment process (Deng, 2010), It should be noted that the elastic parameters of rocks are reduced during an acid-rock reaction; however, these changes cannot yet be described quantitatively using a unified theory (Joel and Pournik, 2011). Some scholars found that the elasticity modulus of carbonate rock declined nearly 24%–30% after soaking in 15%–20% gel acid (He and Guo, 2013). In this research, attempts were made to estimate the effects of closure stress on the acid-etched fracture conductivity. All reaction conditions were the same, and the elastic moduli of the cores declined 20% (for simplification).

2.3. Digitization of the acid-etched fracture

The original 3-D clouds of points were acquired from the high-resolution noncontact 3D morphology scanner (Fig. 2), which can achieve a precision up to ±0.1 mm in the x and y directions and 0.01 mm in the z direction. Then, the data were processed and 3D models of the fracture surfaces etched with gel acid were reconstructed (Fig. 3), and contour maps (Fig. 4) of the crack spacing were plotted by superimposing two fracture surfaces. Fracture spacing distribution can be observed directly from Fig. 4. The color bars represent rough surfaces height and crack spacing separately in Figs. 3 and 4.

2.4. Experimental research on acid-etched fracture conductivity

Fig. 5 is the schematic illustration of the acid-etched fracture conductivity experimental apparatus. It is possible to achieve displacement pressure in the vicinity of 40 MPa, and the closure

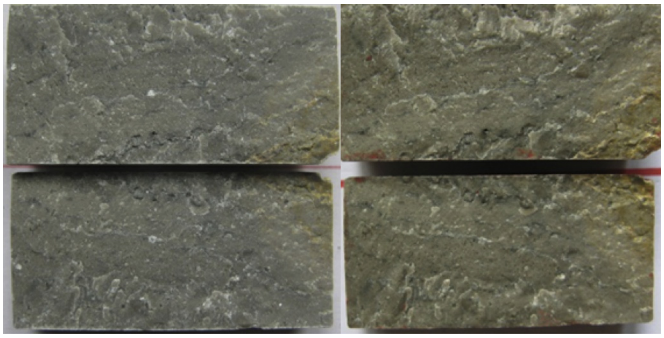

Fig. 1. Fracture surface morphology before (right) and after (left) the reaction with gelled acid.