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Comparison study on the accuracy and efficiency of the four forms of hydraulic equation of a natural gas pipeline based on linearized solution

Peng Wang, Bo Yu*, Yajun Deng, Yu Zhao

National Engineering Laboratory for Pipeline Safety, Beijing Key Laboratory of Urban Oil and Gas Distribution Technology, China University of Petroleum, Beijing, 102249, China

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ABSTRACT

Linearization technique is an efficient and fast method for the natural gas pipeline simulation. In order to determine the optimal selection of basic variables which have advantages on both the accuracy and efficiency for linearization method, the present paper systematically researched on the four forms of hydraulic equations according to various selections of the basic variables for solving the hydraulic equations: pressure and mass flux combination ($F1(p,m)$), pressure and velocity combination ($F2(p,w)$), density and mass flux combination ($F3(\rho,m)$), density and velocity combination ($F4(\rho,w)$), and the efficiency as well as the accuracy is compared based on their linearizations. Through analyzing the numerical results, it can be concluded that $F4(\rho,w)$ is the most efficient and easiest form to implement on a computer for solving hydraulic equations as natural gas simulation is based on linearization technique, while the calculation using $F1(p,m)$ turns out to be the slowest one.

1. Introduction

As a clean and efficient fossil energy, the use of natural gas has gradually gotten more and more attention in the world and the long-distance pipelining is no doubt the most convenient and economic way for natural gas transportation. Depending on the season, weather, number of users, emergencies and other uncertain factors, the flow of natural gas in the pipeline generally is unsteady and transient. Accurate prediction and real-time monitoring of the flow parameters is the premise to guarantee the safe operation of the natural gas pipeline. Under this circumstance, the role of pipeline simulation is of great importance.

The simulation of natural gas pipeline is composed of two parts, hydraulic simulation and thermodynamic simulation. Hydraulic simulation aims to solve the continuity equation and momentum equation which are the so-called hydraulic equations to obtain the hydraulic parameters such as pressure and flow rate. And thermodynamic simulation is to solve the energy equation, also known as the thermodynamic equation, to get the temperature, enthalpy and other thermodynamic parameters. Because the hydraulic parameters are essential for describing natural gas flow inside the pipeline, historically, researchers only calculated the hydraulic equations (Wylie et al., 1974; Kiuchi, 1994). For the non-isothermal pipelines where the solution of thermodynamic equations is required, the thermodynamic hydraulic decoupling solving technology can be employed in these cases to decompose the simulation process into hydraulic simulation and thermodynamic simulation, which means the hydraulic equations are first to be solved to get the hydraulic parameters, like pressure and flow rate, then the thermodynamic equations are calculated on the basis of hydraulic parameters (Helgaker and Ytrehus, 2012; Barley, May. 2012). Given the importance of the hydraulic equations in pipeline simulation, it is crucial to find efficient methods for accelerating the simulation process.

In 1979, Luskin (Luskin, 1979)introduced an efficient and fast linearization method of solving the asymmetric and nonlinear hyperbolic system, thus providing an algorithm for the efficient natural gas pipeline simulation. This method expands the nonlinear terms of partial differential equation in a Taylor series keeping the 1st order term and drops all higher order terms, thus transform the nonlinear equations into linear equations. After actual applications of this method in natural gas pipeline simulation, the results show

* Corresponding author.
E-mail address: yubobox@vip.163.com (B. Yu).

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Nomenclature

d	inner diameter of pipeline, m
g	gravity acceleration, m/s^2
m	mass flux, kg/s
p	pressure, Pa
t	time, s
t_c	CPU time, s
t_{pc}	percentage of CPU time consumed by single module,%
w	velocity, m/s
x	spatial coordinate, m
A	flow area, m^2
Dt_c	the difference in CPU time, s
Dt_{pc}	percentage of CPU time difference by single module, %
U	general hydraulic variable
T	temperature, K

Greek symbols

ρ	density, kg/m^3
θ	pipeline inclination angle, rad
λ	friction coefficient
ϕ	general variable
ε	maximum relative error
Δx	distance between adjacent nodes, m

Superscript

n	nth time layer

that the linearization method can greatly speed up the hydraulic simulation mean while guarantee high calculation accuracy.

Because the natural gas pipeline simulation involves many physical variables, such as pressure, density, flow rate and flow velocity, different selection of parameters as the basic variables can result in various forms of hydraulic equations, such as pressure and mass flux combination (Kiuchi, 1994; Helgaker and Ytrehus, 2012; Liu et al., 2009; Alamian et al., 2012), pressure and velocity combination (Chaczykowski, 2009; Osiadacz and Chaczykowski, 2010), pressure and velocity combination (Ouchiha et al., 2012), density and mass flux combination (Zhou and Adewumi, Oct. 1995), and density and velocity combination (Nouri-Borujerdi, 2011; Modisette, 2012). In natural gas pipeline simulation using the linearization technique, different combination of the variables can lead to the following two differences: The first one is that the coefficients of the obtained equation hold various compact degrees. The second difference lies in the solving process of non-basic variables, that is the coupled solving of pressure and density may be an explicit process of directly substituting or an implicit process of iterating. However, whether these differences have impact on the computation accuracy or speed of the linearization approach has never been investigated, which makes it confusing to select the right hydraulic equation used for fast simulation of natural gas pipeline system.

In order to determine the optimal selection of basic variables which have advantages on both the accuracy and efficiency for linearization method, the present paper systematically researched on the four forms of hydraulic equations: pressure and mass flux combination ($F1(p,m)$), pressure and velocity combination($F2(p,w)$), density and mass flux combination ($F3(\rho,m)$), density and velocity combination ($F4(\rho,w)$), and investigated their accuracy and efficiency in terms of discrete equations and numerical results.

2. Four forms of hydraulic equations

For an infinitesimal natural gas pipeline segment, the differential expression of the hydraulic equations can be written based on the conversation law of mass or momentum as follows.

Continuity equation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho w)}{\partial x} = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho w w)}{\partial x} + \frac{\partial p}{\partial x} = -\frac{\lambda}{2}\frac{\rho w|w|}{d} - \rho g \sin\theta \tag{2}$$

With the following equations,

$$\text{State equation } p = p(\rho(x,t), T(x,t)) \tag{3}$$

Derivation rule of composite functions:

$$\frac{\partial \rho}{\partial t} = \left(\frac{\partial \rho}{\partial p}\right)_T \frac{\partial p}{\partial t} + \left(\frac{\partial \rho}{\partial T}\right)_p \frac{\partial T}{\partial t} \tag{4}$$

$$\frac{\partial \rho}{\partial x} = \left(\frac{\partial \rho}{\partial p}\right)_T \frac{\partial p}{\partial x} + \left(\frac{\partial \rho}{\partial T}\right)_p \frac{\partial T}{\partial x} \tag{5}$$

$$\frac{\partial p}{\partial x} = \left(\frac{\partial p}{\partial \rho}\right)_T \frac{\partial \rho}{\partial x} + \left(\frac{\partial p}{\partial T}\right)_\rho \frac{\partial T}{\partial x} \tag{6}$$

Fundamental equations of thermodynamic functions (Atkins and de Paula, 2010) :

$$\left(\frac{\partial p}{\partial \rho}\right)_T = \frac{1}{(\partial \rho/\partial p)_T} \tag{7}$$

$$\left(\frac{\partial \rho}{\partial T}\right)_p = -\frac{(\partial p/\partial T)_\rho}{(\partial p/\partial \rho)_T} \tag{8}$$

$$\text{Mass flux}: \quad m = A\rho w \tag{9}$$

We can transform Eq. (1) and Eq. (2) into the system of equations

$$\frac{\partial U}{\partial t} + B\left(U\right)\frac{\partial U}{\partial x} = F\left(U\right) \tag{10}$$

The corresponding coefficients are presented in Table 1.

3. Discretization and solution based on linearized solution

The present paper firstly linearized the hydraulic equations of the natural gas pipeline, then discretized the equations with the fully implicit scheme which is unconditionally stable. After obtaining the linearized algebraic equations, the TDMAmethod can be employed to get the final solutions.