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Wave propagation through an inhomogeneous slab sandwiched by the piezoelectric and the piezomagnetic half spaces

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ABSTRACT

Wave propagation through a gradient slab sandwiched by the piezoelectric and the piezomagnetic half spaces are studied in this paper. First, the secular equations in the transverse isotropic piezoelectric/piezomagnetic half spaces are derived from the general dynamic equation. Then, the state vectors at piezoelectric and piezomagnetic half spaces are related to the amplitudes of various possible waves. The state transfer equation of the functionally graded slab is derived from the equations of motion by the reduction of order, and the transfer matrix of the functionally gradient slab is obtained by solving the state transfer equation with the spatial-varying coefficient. Finally, the continuous interface conditions are used to lead to the resultant algebraic equations. The algebraic equations are solved to obtain the amplitude ratios of various waves which are further used to obtain the energy reflection and transmission coefficients of various waves. The numerical results are shown graphically and are validated by the energy conservation law. Based on the numerical results on the fives of gradient profiles, the influences of the graded slab on the wave propagation are discussed. It is found that the reflection and transmission coefficients are obviously dependent upon the gradient profile. The various surface waves are more sensitive to the gradient profile than the bulk waves.

1. Introduction

Piezoelectric/piezomagnetic materials are a class of functionally composite materials. These materials are referred to as smart or intelligent materials because they can provide helpful feedback on the internal deformation state of the material/structure. Due to the wide application of piezoelectric and piezomagnetic materials in the sensor, actuator and surface wave devices, the wave propagation in piezoelectric and piezomagnetic materials has been studied extensively in the past decades. Alshits et al. [1,2] contributed many efforts to this field. They solved various reflection problems of acoustic-electric coupled waves in a semi-infinite piezoelectric medium with consideration of the metallized and non-metallized surfaces. Different from the purely elastic case, the acoustic waves propagating in piezoelectric solids are always accompanied by the waves of the quasi-static electric fields due to the electromechanical coupling effect. So these waves are also called acoustic-electric coupled waves. From theoretical standpoints, Shuvalov and Lothe [23] demonstrated the reciprocity properties of reflection–transmission on a welded and electrically open interface between two piezoelectric media. Furthermore, they developed an approach to prove the reciprocity properties under various boundary conditions for the reflection–transmission problems. Pang et al. [20] studied the reflection and transmission of an elastic wave at the interface between the piezoelectric material ($BaTiO_3$) and the piezomagnetic material ($CoFeO_4$). It is found that there exist four independent basic waves in semi-infinite piezoelectric or piezomagnetic anisotropic elastic media. Li and Wang [11] studied the propagation and localization of elastic waves in disordered periodic layered piezoelectric composite structures with the mechanic–electric coupling effect taken into consideration. Rodríguez-Ramos et al. [21] studied the behavior of transmission coefficients for shear horizontal (SH) wave propagation with oblique incidence in piezocomposite layered systems. The effects of the frequency, incidence angle and piezoelectric volume fraction on the transmission coefficient are discussed. Guo and Wei [7,8] studied also the effects of initial stress and the dielectrically imperfect interface on the reflection and transmission waves at the interface between two different piezoelectric half spaces.

The functionally graded materials have a lot of superiorities over the homogeneous materials, specially, in the thermal shielding and avoiding stress concentration. Hence, functionally graded

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materials have been widely used in spaceflight, aerospace, and electronic engineering fields. To improve the efficiency of the surface wave devices, the gradient piezoelectric/piezomagnetic materials are also considered for their application in the surface wave devices. The wave propagation in gradient piezoelectric/piezomagnetic materials has thus attracted many attentions. Li et al. [12] have studied the Love wave in the semi-infinite medium where the covering layer is a gradient piezoelectric layer and the material property changes by the index rule. Liu and Tani [14–16], have studied surface waves in the gradient piezoelectric plates. It is found that waves of lower order modes in the gradient piezoelectric plates appear as surface waves for large wave numbers and that a strong surface wave is excited on the softer surface of the gradient piezoelectric plate. Scattering problems of anti-plane shear waves by a crack in gradient piezoelectric/piezomagnetic materials were studied by Jun [17]. By using the Fourier transform method, the problem can be solved with the help of a pair of triple integral equations. These equations are solved by using the Schmidt method. Then, the relations among the electric field, the magnetic field, and the dynamic stress field near the crack tips can be obtained. The dynamic stress for a circular hole in gradient piezoelectric/piezomagnetic materials subject to shear waves was studied by Wang et al. [24]. Analytical solutions of the dynamic stress concentration factor, and electric and magnetic fields around the hole are presented. The wave fields, electric and magnetic potentials are expanded using a wave function expansion method. Lan and Wei [10] studied the band gaps of a laminated piezoelectric/piezomagnetic phononic crystal with graded interlayer, the influences of the graded interlayer with different gradient profiles on the band gap of a laminated piezoelectric/piezomagnetic phononic crystal are discussed. Their investigations reveal that the band gaps at the higher frequency range are more sensitive to the gradient interlayers. Fomenko et al. [6] studied the in-plane elastic wave propagation and band-gaps in layered functionally graded phononic crystals and gave a criterion for stop-band calculations using eigenvalues of the transfer matrix for a unit-cell. Wu et al. [25] studied the elastic wave band gaps of one-dimensional phononic crystals with functionally graded materials by using the spectral finite elements and transfer matrix methods. Although, the wave propagation in the gradient piezoelectric or piezomagnetic materials and the band-gaps in layered functionally graded phononic crystals were studied in the above-mentioned literatures, it is often based on approximating the gradient layer by many homogeneous sublayers. The introduction of the additional interfaces is inevitable and may lead to inaccuracy. Accordingly, the direct integral method is wanted.

In this paper, wave propagation through a gradient magneto-electro-elastic slab sandwiched by the piezoelectric and piezomagnetic half spaces is studied. The material properties of the gradient magneto-electro-elastic slab change continuously from the piezoelectric material to the piezomagnetic material. In order to overcome the difficulty resulting from the inhomogeneity and anisotropy of the slab, the state vector is introduced and the state transfer equation of the functionally graded slab is derived from the equations of motion by reduction of order. The transfer matrix of the functionally graded slab is obtained directly by solving the state transfer equation with the spatial-varying coefficient. Compared with the method approximating the gradient slab by a system of homogeneous sublayers, the introduction of additional interfaces is avoided. The continuous interface conditions lead to the resultant algebraic equations from which the amplitude ratios of various waves and further the energy reflection and transmission coefficients of various waves are obtained. These numerical results are validated by the energy flux conservation.

2. State vectors in the piezoelectric and piezomagnetic half spaces

The constitutive equation of magneto-electro-elastic solid can be written as [22]

$$\sigma_{ij} = c_{ijkl}S_{kl} - e_{kij}E_k - q_{kij}H_K, \tag{1a}$$

$$D_i = e_{ikl}S_{kl} + \varepsilon_{ik}E_k + \alpha_{ik}H_k, \tag{1b}$$

$$B_i = q_{ikl}S_{kl} + \alpha_{ik}E_k + \mu_{ik}H_k, \tag{1c}$$

where σ_{ij} and S_{kl} are the stress and strain tensor. E_k and D_i are the electric field and electric displacement. H_k and B_i are the magnetic field and magnetic induction. c_{ijkl}, e_{ikl}, q_{ikl}, μ_{ik} and ε_{ik} are the elastic, piezoelectric, piezomagnetic, magnetic permeability and dielectric constant, respectively and the α_{ik} is magneto-electric coupling coefficient. Let the z-axis be the poling direction and the material is assumed to be transversely isotropic in the oxy coordinate plane. Then, Eq. (1) reduces to

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{zx} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5(c_{11}-c_{12}) \end{bmatrix} \cdot \begin{Bmatrix} S_{xx} \\ S_{yy} \\ S_{zz} \\ 2S_{yz} \\ 2S_{zx} \\ 2S_{xy} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} - \begin{bmatrix} 0 & 0 & q_{31} \\ 0 & 0 & q_{31} \\ 0 & 0 & q_{33} \\ 0 & q_{15} & 0 \\ q_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix}, \tag{2a}$$

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix} \cdot \begin{Bmatrix} S_{xx} \\ S_{yy} \\ S_{zz} \\ 2S_{yz} \\ 2S_{zx} \\ 2S_{xy} \end{Bmatrix} + \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \cdot \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} + \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{bmatrix} \cdot \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix}, \tag{2b}$$

$$\begin{Bmatrix} B_x \\ B_y \\ B_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & q_{15} & 0 \\ 0 & 0 & 0 & q_{15} & 0 & 0 \\ q_{31} & q_{31} & q_{33} & 0 & 0 & 0 \end{bmatrix} \cdot \begin{Bmatrix} S_{xx} \\ S_{yy} \\ S_{zz} \\ 2S_{yz} \\ 2S_{zx} \\ 2S_{xy} \end{Bmatrix} + \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{bmatrix} \cdot \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} + \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{11} & 0 \\ 0 & 0 & \mu_{33} \end{bmatrix} \cdot \begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix}, \tag{2c}$$