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Coupled circuit based representation of piezoelectric structures modeled using the finite volume method

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ABSTRACT

This paper presents the methodology of generating a corresponding electrical circuit for a simple piezoelectric plate modeled with the finite volume method. The corresponding circuit is implemented using a circuit simulation software and the simulation results are compared to the finite volume modeling results for validation. It is noticed that both, the finite volume model and its corresponding circuit, generate identical results. The results of a corresponding circuit based on the finite volume model are also compared to the results of a corresponding circuit based on a simplified analytical model for a long piezoelectric plate, and to finite element simulation results for the same plate. It is observed that, for one control volume, the finite volume model corresponding circuit and the simplified analytical model corresponding circuit generate close results. It is also noticed that the results of the two corresponding circuits are different from the best approximation results obtained with high resolution finite element simulations due to the approximations made in the simplified analytical model and the fact that only one finite volume was used in the finite volume model. The implementation of the circuit can be automated for higher order systems by a program that takes as an input the matrix of the system and the forcing function vector, and returns a net list for the circuit.

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1. Introduction

Circuit simulation programs that can handle a large number of circuit components are readily available on the market (PSPICE, Multisim, CircuitLab, SIMetrix, PLECS). These programs are fast and accurate as a result of improved algorithms, more efficient programming procedures, and improved hardware resources. The use of circuits presumes that the problem is of finite order. Dependent sources are used to describe coupling, for example coupling between an electrical and mechanical field (electro-mechanical circuit) or coupling between an electrical field and a thermal field (electro-thermal circuit). Nonlinearities can also be incorporated in terms of dependent sources.

In contrast, most physical problems are cast in the form of coupled partial differential equations for which the discretized system has in principle an infinite number of state variables and coupled differential equations. A higher degree of accuracy requires greater discretization. However, practically speaking, it is not necessary to have perfect accuracy therefore some form of model order reduction process is performed in order to obtain a tractable set of equations and these equations can be then visualized in terms of coupled circuits if desired.

In many cases, researchers develop a circuit representation of a problem using simplifications but these cannot be traced back to any rigorous process. For example, the analysis of electromechanical systems with circuits have been performed by McDermott, Zhou, and Gilmore in [1]. Instead of performing time-stepped field simulations, which take a long time, the authors proposed running system simulations of a behavioral model extracted from a set of parametric finite element solutions.

Ekinci and Atalar in [2] used circuit simulation programs to solve structural mechanical problems with equivalent circuits. In their paper they apply the finite element formulation to a mechanical problem and obtain a set of equations which are treated as if they were the set of equations of an equivalent circuit that contains linear elements such as capacitors, inductors, and controlled sources. The equivalent electrical circuit obtained from this set of equations is then solved using a general purpose circuit simulator.

Reference [3] presents the underlaying theory of constructing an equivalent electrical circuit describing the electromechanical behavior of a piezoelectric bimorph beam. By using the equation of motion and the electrical and mechanical boundary conditions,

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an electromechanical impedance matrix of the bimorph beam is obtained. This naturally leads to the equivalent circuit representation of the bimorph beam.

The authors in [4] use the analogy in dynamic equations between a mechanical and an electrical system to generate the equivalent electric circuit model for a turbine-generator system. First, an equivalent electrical circuit model is developed for a turbine-generator mechanical model. Second, an equivalent electrical interface circuit model is derived from the synchronous generator's dynamic equations and the electromagnetic torque equation. Finally, an integrated circuit model is obtained by using the electric interface circuit to connect the circuit model of the synchronous generator and the equivalent electrical circuit model of turbine-generator mechanism. The integrated circuit model is used for analyzing the interactions between the mechanical and electrical systems for a steam turbine generator unit.

References [5,6] present a new approach for converting mechanical and electromechanical elements to equivalent circuits for MEMS devices. Reference [5] presents a method to obtain equivalent circuits for MEMS devices starting from the Lagrangian L of the comb-drive actuator function of the velocity v, displacement x, and charge q. Reference [6] proposes equivalent circuit expressions for mechanical connection of MEMS devices modeled with the method in [5] by using the graph theory for mechanical connections.

As mentioned previously, real physical models have to be extracted from a discretized version of the partial differential equations that describe the system. The finite element method is one of the most commonly used approaches for solving the discretized problem since it allows complex geometries to be modeled with relative ease. However, reducing these equations into a circuit based form and interfacing them with an external controller is not a transparent process and the conservation of flux across boundaries is not necessarily observed. A number of simulation programs allow the coupling of a finite element model of a process to an external circuit but there is no intuition as to how to design a controller to satisfy control requirements. On the other hand, most practical devices are relatively simple geometric structures and hence an alternative means of discretizing these systems is to use the Finite Volume Method (FVM) which has some attractive features that are described next.

It has been shown recently in [7,8] that using the FVM to model piezoelectric devices has the following strengths:

- The FVM Ordinary Differential Equations (ODE) can be interpreted directly in terms of coupled circuits that represent averages of a specific parameter over a volume element in the piezoelectric system [9]; divergence law and Stoke's theorem can be applied directly. This makes it easier to interface the FVM model of the piezoelectric system with control circuits where the control parameters or quantities being measured are also in terms of averaged quantities.
- The FVM works easily with surface integrals making it easier to deal with phenomena that occur at the boundary between two different materials [8]. Therefore, this method may be more suitable to model an ultrasonic motor because the operating principle of the motor is based on the friction mechanism that takes place at the common contact boundary between the stator and the rotor.

This paper proposes a method of generating corresponding electrical circuits of piezoelectric devices based on their modeling with the finite volume method. The corresponding electrical circuits are generated based on the ordinary differential equations and algebraic equations of the finite volume method model of the piezoelectric device. The paper concludes with a modeling example for a simple piezoelectric plate and a comparison with the results generated by a corresponding circuit based on a simplified analytical model of a long piezoelectric plate.

2. The corresponding circuit representation

Modeling of a piezoelectric device with the finite volume method is shown in [7]; a simple piezoelectric plate modeled using the finite volume method results in a system of coupled differential equations, which describe the dynamics of the control volumes, and algebraic equations, which describe the boundary conditions. The system of equations was solved in a numerical simulation program such as Matlab and displacement and eigenfrequency of the plate were calculated. Having a model of a piezoelectric device in the form of a system of differential and algebraic equations is useful for the calculation of displacements and eigenfrequencies but it is not very useful when it has to be used in a control scheme. The control schemes are usually implemented with electric and electronic devices, therefore, it would be useful to have a model of the piezoelectric device in the form of a circuit or a state space equivalent form. Thus, the model of the piezoelectric device could be easily interfaced with the control circuit.

In this paper, the generation of the corresponding circuit for a piezoelectric device using the FVM is demonstrated for a simple piezoelectric plate shown in Fig. 1 and modeled with the FVM in [7]. The corresponding circuit for each FVM ODE is generated by starting with Eqs. (37)–(39) in [7]. These equations model the dynamics of the (average) displacements u_P, v_P, and w_P of a volume in the x, y and z directions. In this paper it is shown only the processing of equation (37), equations (38) and (39) being processed in a similar way. In order to generate a corresponding circuit with components that directly duplicate the mechanical system, it is useful to write Eq. (37) in the form of the second law of dynamics. This is obtained by multiplying Eq. (37) in [7] by the mass of the control volume $\Delta V\rho$ as shown in Eq. (1).

$$\begin{aligned}\Delta V\rho\frac{d^2u_P}{dt^2} &= -P_1'u_P+\mathcal{E}_1'u_E+W_1'u_W+N_1'u_N+S_1'u_S+F_1'u_F+R_1'u_R\\ &+B_{11}'(v_N-v_S)+B_{12}'(v_{NE}-v_{SE})-B_{13}'(v_{NW}-v_{SW})\\ &+B_{14}'(v_E-v_W)+B_{15}'(v_{NE}-v_{NW})-B_{16}'(v_{SE}-v_{SW})\\ &+B_{17}'(w_F-w_R)+B_{18}'(w_{FE}-w_{RE})-B_{19}'(w_{FW}-w_{RW})\\ &+B_{110}'(w_E-w_W)+B_{111}'(w_{FE}-w_{FW})-B_{112}'(w_{RE}-w_{RW}) \quad (1)\end{aligned}$$

In this equation, the coefficients with prime "′" are obtained by multiplying the coefficients without prime by $\Delta V\rho$. For example, $\mathcal{E}_1' = \mathcal{E}_1\Delta V\rho$. Eq. (1) is the equation for the second law of dynamics where the left hand side represents the mass multiplied by acceleration and the right hand side is the elastic force generated by a series of springs. The coefficients of the displacements in the right hand side are the springs' stiffness and have the unit of N/m. One is interested in determining the value for the displacement u_P inside the current volume and the equation is divided by P_1' as shown in Eq. (2).

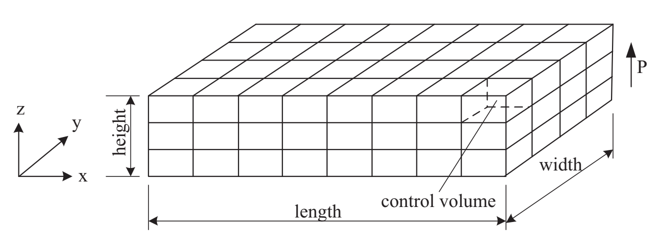

Fig. 1. Piezoelectric plate.

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