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Advances in Space Research 58 (2016) 1201–1207

ADVANCES IN SPACE RESEARCH
(a COSPAR publication)

www.elsevier.com/locate/asr

Study of instability of the nonelectroneutral current sheets in quasilinear approximation

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Received 15 March 2016; received in revised form 1 June 2016; accepted 10 June 2016
Available online 25 June 2016

Abstract

The techniques of investigation of low-frequency instabilities of the current sheet in quasi-linear approximation with due regard to the effect of plasma polarization is described. This makes it possible to study the mechanism of the reverse impact of the developing instability modes of nonelectroneutral current sheet on the background distribution function.

Evolution equations of the equilibrium distribution function of the plasma current sheet and the perturbation of the electromagnetic field are derived. Algorithms for solution of evolution equations are developed. A dispersion curve for the plasma wave with damping decrement propagating along the magnetic field is derived.

Keywords: Vlasov' equations; Weak turbulent plasma; Quasi-linear approximation; Nonelectroneutral current sheet; Low-frequency instabilities; Transformation of the basic distribution function

1. Introduction

To describe the steady state of a current sheet, Harris offered in due time the distribution function chosen in such a way that it allows passing into a coordinate system where the electric field is zero (Harris, 1962). Further, Coppi et al. (1966) investigated the stability of the sheet, and they arrived at the conclusion that the development of the tearing-instability which enables reconnection of the magnetic field lines in the magnetoneutral plane is possible under certain conditions. Those two papers became a basis for most of the further research of the dynamics of a magnetotail current sheet and energetics of magnetospheric substorms.

Following theoretical studies (Brittnacher et al., 1994, 1998; Lembege and Pellat, 1982; Pellat et al., 1991) showed that inclusion of the component of the magnetic field, that is normal to the sheet, changes the power balance of the system and the spontaneous development of the instability becomes energetically unfavorable. One of the directions of research of the dynamics of current sheets now is searching for physical processes that would facilitate the development of tearing and other instabilities. The instability of the current sheets are studied under the influence of such disturbances as bending, balloon, combined, drift mode (Sitnov and Lui, 1999; Buchner and Kuska, 1999; Daughton, 1998; Kuznetsova et al., 2001; Sitnov et al., 1999; Silin et al., 2002; Zelenyi et al., 2002; Mingalev et al., 2007). Investigation of the problem in a non-linear approach allows us to answer the question to what point can grow the initial perturbations, and how they affect the initial steady state plasma. The paper (Camporeale et al., 2004) investigates the problem of starting of reconnection of the magnetic field lines, and it may be inferred that the development of low-hybrid drift instability can contribute to solving this problem.

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http://dx.doi.org/10.1016/j.asr.2016.06.016
0273-1177/

Plasma polarization effects of sharply nonuniform plasma structures, like the current sheets and the magnetopause, on their equilibrium and stability in the linear approximation are investigated in our papers (Lyahov and Neshchadim 2012, 2013, 2014a, 2014b).

However, the linear theory of instability of the current sheet makes it possible to answer only the question about the development of small perturbations at the initial stage, when their amplitudes are small. Study of the problem in a nonlinear approximation allows us to answer the question to what point can grow the initial perturbations, and how they affect the initial steady state of plasma

2. Basic equations

We use a technique of examination of oscillations in a quasilinear approximation, applicable to weak turbulent plasma (Alexandrov et al., 1988). The criterion for the condition of weak plasma turbulence is the relation

$$\frac{W}{n\varepsilon} \ll 1. \tag{1}$$

Here, $n\varepsilon$ is the thermal energy of the particles, $W = \sum_k W_k = \sum_k \frac{\varepsilon_0 E_k^2}{2}$ – the energy of plasma oscillations per unit volume, E_k – intensity of electric field of ostsilyation, n – the concentration of particles. That is, the energy of the oscillations resulting from the development of instability is small compared to the thermal energy of the plasma particles.

We consider only the longitudinal (potential) plasma oscillations. These oscillations are described by a nonlinear system of Vlasov' equations

$$\frac{\partial f_\alpha}{\partial t} + \vec{v}\frac{\partial f_\alpha}{\partial \vec{r}} + e_\alpha\{\vec{E} + [\vec{v}\vec{B}_0]\}\frac{\partial f_\alpha}{\partial \vec{P}_\alpha} = 0, \tag{2}$$

$$div\vec{E} = \frac{\rho}{\varepsilon_0}, \tag{3}$$

where

$$\rho = \sum_\alpha e_\alpha \int f_\alpha d\vec{P}. \tag{4}$$

The distribution function is represented as a sum of large slow and small fast parts:

$$f_\alpha(\vec{P}, t) = f_{0\alpha}(\vec{P}, \mu t) + f_{1\alpha}(\vec{P}, t). \tag{5}$$

Coefficient μ mean a slow time dependence.

Here, according to the condition,

$$f_{0\alpha}(\vec{P}, \mu t) \ll f_{1\alpha}(\vec{P}, t).$$

We expand the fast part of distribution function and the electromagnetic field in the plasma to Fourier series:

$$f_{1\alpha}(\vec{P}, t) = \sum_l \mathrm{Re}\left[f_{1\alpha l}(\omega, \vec{k}, \mu t)\exp(-i\omega t + i\vec{k}\vec{r})\right],$$

$$\delta\vec{E}(\vec{r}, t) = \sum_l \mathrm{Re}\left[\delta\vec{E}_l(\omega, \vec{k}, \mu t)\exp(-i\omega t + i\vec{k}\vec{r})\right], \tag{6}$$

$$\delta\phi(\vec{r}, t) = \sum_l \mathrm{Re}[\delta\phi_l(\omega, \vec{k}, \mu t)\exp(-i\omega t + i\vec{k}\vec{r})].$$

Substituting of the expansions (5), (6) in the initial Eq. (2) and averaging out of the fast oscillations allows obtaining the equations for the slow and fast parts of the distribution function:

$$\frac{\partial f_{0\alpha}}{\partial \mu t} + e_\alpha\left\langle \vec{E}\frac{\partial f_{1\alpha}}{\partial \vec{P}_\alpha}\right\rangle + e_\alpha[\vec{v}\vec{B}_0]\frac{\partial f_{0\alpha}}{\partial \vec{P}_\alpha} = 0, \tag{7}$$

$$\frac{\partial f_{1\alpha}}{\partial t} + \vec{v}\frac{\partial f_{1\alpha}}{\partial \vec{r}} + e_\alpha\vec{E}\frac{\partial f_{0\alpha}}{\partial \vec{P}_\alpha} + e_\alpha[\vec{v}\vec{B}_0]\frac{\partial f_{1\alpha}}{\partial \vec{P}_\alpha} = 0. \tag{8}$$

Using Fourier expansion of the electromagnetic field (6) and taking into account the potentiality of electric field $\vec{E} = -grad\delta\phi$ one can obtain from the Eq. (8) the following expression:

$$f_{1\alpha l} = e_\alpha \sum_{n,m} \frac{J_n\left(\frac{k_\perp v_\perp}{\Omega_\alpha}\right) J_m\left(\frac{k_\perp v_\perp}{\Omega_\alpha}\right)}{\omega - k_x v_x - n\Omega_\alpha}\delta\phi_l \cdot \left(k_x\frac{\partial}{\partial p_x} + \frac{n\Omega_\alpha}{v_\perp}\frac{\partial}{\partial p_\perp}\right) f_{\alpha 0}\cdot \exp[i(m-n)\Phi]. \tag{9}$$

Substituting of this solution in the Eq. (7) and averaging over time one give finally an equation for the slow part of the distribution function in a quasi-linear approximation:

$$\begin{aligned}\frac{\partial f_{0\alpha}(v_x, v_\perp, \Phi, \mu t)}{\partial t} &= -\frac{e_\alpha^2}{2}\sum_l\sum_n \delta\phi_l(\omega, \vec{k}, \mu t)\cdot\delta\phi_l^*(\omega, \vec{k}, \mu t)\\ &\times \frac{1}{m_\alpha}\left(k_x\frac{\partial}{\partial v_x} + \frac{n\Omega_\alpha}{v_\perp}\frac{\partial}{\partial v_\perp}\right)\cdot J_n^2\left(\frac{k_\perp v_\perp}{\Omega_\alpha}\right)\\ &\cdot \mathrm{Im}\left(\frac{1}{\omega - k_x v_x - n\Omega_\alpha}\right)\\ &\cdot\frac{1}{m_\alpha}\left(k_x\frac{\partial}{\partial v_x} + \frac{n\Omega_\alpha}{v_\perp}\frac{\partial}{\partial v_\perp}\right) f_{0\alpha}(v_x, v_\perp, \Phi, \mu t).\end{aligned} \tag{10}$$

At the same time, the evolution of the Fourier-amplitude of the electric field perturbation satisfies the equation:

$$\frac{\partial|\delta\vec{E}_l(\omega, \vec{k}, \mu t)|^2}{\partial t} = 2\delta_l|\delta\vec{E}_l(\omega, \vec{k}, \mu t)|^2, \tag{11}$$

where

$$\begin{aligned}\delta_l(\omega, \vec{k}, \mu t) = \frac{1}{2}\sum_\alpha\sum_n \frac{e_\alpha^2 \mathrm{Re}\omega m_\alpha^3}{\varepsilon_0 k^2}\int_{-\infty}^{+\infty} dv_x \int_0^{+\infty} v_\perp dv_\perp\\ \times \int_0^{2\pi} d\Phi\cdot \mathrm{Im}\left(\frac{1}{\omega - k_x v_x - n\Omega_\alpha}\right)\\ \cdot J_n^2\left(\frac{k_\perp v_\perp}{\Omega_\alpha}\right)\\ \cdot\frac{1}{m_\alpha}\left(k_x\frac{\partial f_{\alpha 0}}{\partial v_x} + \frac{n\Omega_\alpha}{v_\perp}\frac{\partial f_{0\alpha}(v_x, v_\perp, \Phi, \mu t)}{\partial v_\perp}\right).\end{aligned} \tag{12}$$