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On–off intermittency and spatiotemporal chaos in three-dimensional Rayleigh–Bénard convection

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Abstract

Convective instabilities of viscous conducting fluids play an important role in many physical phenomena in planets and stars. Astrophysical magnetic fields are usually explained in a framework of the dynamo theory, describing transformation of the kinetic energy of a flow into magnetic energy. Therefore, an analysis of purely convective states and their bifurcations, as a control parameter is changed, provides a detailed framework for the subsequent study of magnetic field generation by these states. In this paper, three-dimensional Rayleigh–Bénard convection in the absence of magnetic field is investigated numerically for various values of the Rayleigh number and a fixed Prandtl number (corresponding to its value for convection in the Earth's outer core). On increasing the Rayleigh number, we identified periodic, quasiperiodic, chaotic and hyperchaotic attractors of the convective system and their bifurcations, thereby describing a route to spatiotemporal chaos in the convective system. The occurrence of on–off intermittency in the energy time series is discussed.

Keywords: Rayleigh–Bénard convection; Hyperchaos; Spatiotemporal chaos

1. Introduction

Thermal convection is one of the most efficient and widespread mechanisms of mass and energy transport in fluids, acting in the terrestrial atmosphere as well as in the interior of planets and stars. Solar convection is responsible for the formation of fluid cells observable as granular patterns in the photosphere (Chian and Kamide, 2007). Convective flows with a strong shear in the solar convection zone are believed to be responsible for intensification of the magnetic flux (Brandenburg and Subramanian, 2005). Efficiency of this amplification mechanism relies on some properties of the velocity field, that should be able to stretch, twist and fold the magnetic field lines in such way that magnetic flux is increased (Childress and Gilbert, 1995). Terrestrial magnetic fields are also generated by convective flows of a conducting fluid in the liquid outer core (Rüdiger and Hollerbach, 2004).

Rayleigh–Bénard (R–B) convection refers to the motion of a viscous fluid in a plane horizontal layer heated from below in a gravitational field with a vertical temperature gradient. It has been extensively studied due to the feasibility of both analytical, numerical and experimental treatments (Chandrasekhar, 1961; Bodenschatz et al., 2000). For over a century, R–B convection has been explored for pattern formation in systems outside of equilibrium (Bénard, 1901; Cross and Hohenberg, 1993). It is also the

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simplest framework to explain the formation of convective patterns in astro- and geophysics (Fowler, 2005).

Control parameters in R–B convection are the Rayleigh number, R, measuring the magnitude of the thermal buoyancy force, and the Prandtl number, P, the ratio of kinematic viscosity to thermal diffusivity. Values of these parameters identify properties of the convective flow, therefore regions in the two-dimensional parameter space are used to describe instabilities, pattern formation, symmetry breaking and transition to turbulence in convection.

The dynamical systems theory and the bifurcation theory have been used to explain transition to turbulence in the R–B convection since Edward Lorenz's reduced model (Lorenz, 1963). Its chaotic behavior, characterized by irregular time dynamics, sensitivity to initial conditions and presence of a positive Lyapunov exponent, was a first step towards a dynamical systems description of a turbulent flow. The Lyapunov exponents measure the average exponential rate of growth/shrinkage of initially close trajectories in the phase space. Variating a control parameter of a hydrodynamic system towards turbulent flows, one expects the system to become progressively more irregular in time and space, in a state with more than one positive Lyapunov exponent – *hyperchaotic* state (Rössler, 1979). Hyperchaos was found in generalized Lorenz systems (Zhou et al., 2008; Macek and Strumik, 2014). In a more realistic setup, Paul et al. (2011) detected hyperchaos in a reduced model of the R–B convection, for two-dimensional flows and keeping only 14 complex and 2 real Fourier modes of the solution. Hyperchaotic states were also found in studies of spiral defect chaos using simulations of three-dimensional R–B convection in cylindrical domains (Egolf et al., 2000; Paul et al., 2007; Karimi and Paul, 2012). The spectrum of Lyapunov exponents was used *ibid.* to compute the fractal dimension of the underlying convective state, by this means quantifying the extensive spatiotemporal chaos and studying dependence of the number of dynamical degrees of freedom on the size of the system.

In the present paper, results of a study on transition to hyperchaos in R–B convection in hydrodynamics are reported. The Prandtl number is fixed at $P = 0.3$ and the Rayleigh number is varied as a control parameter. This value of P is interesting for the study of the geodynamo, since in the outer core P is estimated to be between 0.1 and 0.5 (Olson, 2007; Fearn and Roberts, 2007). We follow the works by Podvigina (2006, 2008), where several attractors and bifurcations of the convective system in the same range of parameters were identified. The paper is organized as follows. In Section 2, equations governing the convective system, boundary conditions and numerical methods are presented. In Section 3, attractors of the convective system for increasing values of the Rayleigh number are presented and an observed route to spatiotemporal chaos is discussed. Here, we employ the term spatiotemporal chaos to denote hyperchaos in a spatially extended dynamical system. The conclusions are given in Section 4.

2. The model

A newtonian incompressible fluid is confined between two infinite horizontal planes in a square periodic cell $\mathcal{D} = [0, L]^2 \times [0, 1]$, see Fig. 1. The fluid is uniformly heated from below. Temperatures at the bottom, T_1, and the top, T_2, planes are fixed, with $T_1 > T_2$. Under the Boussinesq approximation, considering the vertical size of the fluid container d as a length scale, the vertical heat diffusion time $\tau_v = d^2/\kappa$ as a time scale, and the vertical temperature gradient δT as a temperature scale, the dynamics of three-dimensional R–B convection in a plane layer is governed, in a dimensionless form, by Chandrasekhar (1961) and Getling (1998) the Navier–Stokes equation,

$$\frac{\partial \mathbf{v}}{\partial t} = P\nabla^2\mathbf{v} + \mathbf{v} \times (\nabla \times \mathbf{v}) + PR\theta\mathbf{e}_z - \nabla p, \tag{1}$$

the heat transfer equation,

$$\frac{\partial \theta}{\partial t} = \nabla^2\theta - (\mathbf{v}\cdot\nabla)\theta + v_z, \tag{2}$$

and the incompressibility condition,

$$\nabla \cdot \mathbf{v} = 0, \tag{3}$$

where $\mathbf{v}(\mathbf{x}, t)$ is fluid velocity, $p(\mathbf{x}, t)$ the modified pressure and $\theta(\mathbf{x}, t) = T(\mathbf{x}, t) - (T_1 + (T_2 - T_1)z)$ is the difference of the temperature and its linear profile. The dimensionless parameters are the Prandtl number P (representing the material properties of the fluid),

$$P = \frac{\nu}{\kappa},$$

and the Rayleigh number R (representing the magnitude of the buoyancy force),

$$R = \frac{\alpha g \delta T d^3}{\nu\kappa},$$

with g representing the gravitational acceleration, ν, κ and α the kinematic viscosity, thermal diffusivity and thermal expansion coefficients, respectively.

The horizontal boundaries (non-deformable and impermeable by the fluid), defined by $z = 0$ and $z = 1$, are held at constant temperatures,

$$T(\mathbf{x}, t)|_{z=0} = T_1, \quad T(\mathbf{x}, t)|_{z=1} = T_2, \quad T_1 > T_2,$$

i.e.,

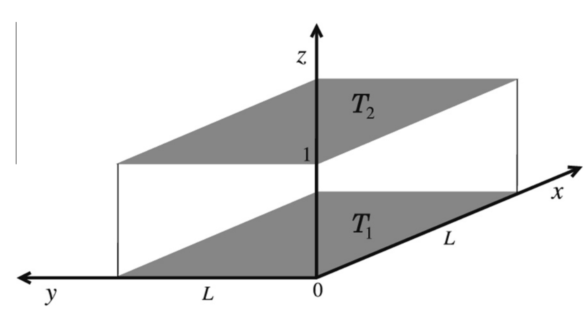

Fig. 1. Computational domain for the horizontal plane layer.