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Mathematical minimum of Geometric Dilution of Precision (GDOP) for dual-GNSS constellations

Yunlong Teng [a,*], Jinling Wang [b], Qi Huang [a]

[a] School of Energy Science and Engineering, University of Electronic Science and Technology of China, Chengdu, Sichuan Province 611731, PR China
[b] School of Civil and Environmental Engineering, University of New South Wales, Sydney, NSW 2052, Australia

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Abstract

Selecting optimal satellites for positioning calculation is a basic problem for the positioning, navigation and timing (PNT) applications with Global Navigation Satellite System (GNSS), and the Geometric Dilution of Precision (GDOP) is a key index to handle this problem. In general, the lower the GDOP values are, the more accurate the PNT solution is. Therefore, the minimum value of GDOP should be pursued. In this paper, we focused on the five-satellite as at least five satellites are required for dual-GNSS constellations. Utilizing the characteristics of matrix partial orders, the mathematical minimum of GDOP in the five-satellite case together with the optimal distribution of the five satellites has been theoretically derived. Furthermore, from a theoretical point of view, the detailed expressions of the impact of different constellational combinations of these satellites on the GDOP have been obtained. The results demonstrated that, for dual-GNSS, even if the geometric distribution of the five satellites is fixed, different constellational combinations of these satellites lead to different values of GDOP. This is different from the single-GNSS case.

Keywords: Global Navigation Satellite System; Geometric Dilution of Precision; Mathematical minimum; Optimal design matrix

1. Introduction

With the modernization of the Global Positioning System (GPS), the revitalization and modernization of the GLONASS and the deployment of Galileo as well as the BDS (Beidou System, and also known as Compass), it is expected that the full operation of multiple constellations is on the horizon. Benefiting from these Global Navigation Satellite System (GNSS) constellations, the number of satellites will be greatly increasing, providing more stable, reliable and accurate services for positioning, navigation and timing (PNT) applications. With the PNT applications using GNSS, the Geometric Dilution of Precision (GDOP) is a quality measure to specify the additional multiplicative affect of measurement error on the positioning accuracy and the timing accuracy. Given the same level of measurement error, the lower GDOP value results in the higher positioning and timing accuracy. Thus, a low value of GDOP is preferred. In addition, the minimum of GDOP (namely, the GDOP optimization) can also be used for selecting satellites for positioning calculation and positioning configuration design (Dempster, 2006; Sharp et al., 2009; Blanco-Delgado and Nunes, 2010; Xue et al., 2014a). Specifically, we should choose the combination of satellites with GDOP as small as possible for positioning calculation. For the positioning configuration design, GDOP minimization is a very crucial factor for the GNSS system design. With large observational freedom, the complete graphs in three-dimensional space with the lowest GDOP value are required for GNSS constellation design

* Corresponding author.
E-mail addresses: ylteng@uestc.edu.cn (Y. Teng), jinling.wang@unsw.edu.au (J. Wang), hwong@uestc.edu.cn (Q. Huang).

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to achieve high coverage and best performance of real-time positioning (Xue and Yang, 2015). Thus, the minimum of GDOP is worthy of further discussion for PNT applications with GNSS constellations.

Aiming at calculating the GDOP minimum in single-GNSS, there are many different algorithms (Fang, 1986; Parkinson et al., 1996; Sairo et al., 2003; Han et al., 2014; Xue and Yang, 2015). These algorithms derived the minimums of GDOP from different points. For instance, Fang (1986) used an eigenvalue approach and found that four satellites which are located in the vertices of a regular tetrahedron give a minimum GDOP of $\sqrt{5/2}$. Using the graph theory, a framework to analytically solve the GDOP optimization with arbitrary observational freedom was established by Xue and Yang (2015). They studied the GDOP minimum and the positioning configuration with the lowest GDOP including constraint and unconstraint. For instance, in terms of unconstraint, there are many graphs with the lowest GDOP, such as cones, regular polyhedrons, Descartes configuration, etc.

Compared with single-GNSS, the dual-GNSS can improve the performance of PNT applications (Wang et al., 2001; Defraigne and Baire, 2011; Angrisano et al., 2013; Odijk and Teunissen, 2013; Teng and Wang, 2014; Teunissen et al., 2014; Paziewski and Wielgosz, 2015). In the process of discussing the GDOP minimum of dual-GNSS, since more than one clock-offset nuisance parameters are involved, it is much harder to analytically solve the GDOP. In our previous studies (Teng et al., 2015), the practical minimum of GDOP for five satellites was discussed. In this paper, we concentrate on the mathematical minimum of GDOP for dual-GNSS constellations.

As there are five unknown parameters to be estimated in dual-GNSS, at least five satellites are required for positioning calculation. The remaining parts of this paper are organized as follows. The definition of GDOP for dual-GNSS constellations is given in Section 2. The specific mathematical minimum of GDOP together with the design matrix in the five-satellite case is theoretically derived in Section 3. In addition, based on optimal distribution of five satellites making the DGDOP reach its mathematical minimum, the impact of different constellational combinations of the five satellites on the DGDOP has also been given in this section. Then the specific reason for this phenomenon has been explained in Section 4. Some concluding remarks and future research directions are given at the end of this paper.

2. Definition of GDOP for dual-GNSS constellations

In this section, the measurement equation for positioning calculation in single-point positioning is introduced firstly. Then the definition of GDOP and the corresponding design matrix for dual-GNSS constellations are given.

When the single GNSS constellation (for example GPS) is used to perform the single-point positioning, the linearization equation for positioning calculation is given by

$$z_1 = \boldsymbol{H}_1 \Delta \boldsymbol{x}_1 \tag{1}$$

In Eq. (1), z_1 represents the measurement vector, $\Delta\boldsymbol{x}_1 = [\Delta\boldsymbol{r}\ c\Delta t_1]^T$ denotes the unknown vector to be estimated and it includes four unknown parameters. In this unknown vector, $\Delta\boldsymbol{r}$ and $c\Delta t_1$ denote the positional parameters in three dimensions and the receiver clock bias, respectively. The matrix $\boldsymbol{H}_1$ is called as the design matrix or the Jacobian matrix of the nonlinear pseudo-distance equations (Xue et al., 2014b), and it captures the receiver-satellite geometry (Teunissen, 1998). The design matrix can be expressed as

$$\boldsymbol{H}_1 = \begin{bmatrix} \boldsymbol{h}_1 & 1 \\ \boldsymbol{h}_2 & 1 \\ \cdots & \cdots \\ \boldsymbol{h}_\alpha & 1 \end{bmatrix} \tag{2}$$

where α denotes the number of satellites, $\boldsymbol{h}_i (i = 1, \ldots, \alpha)$ denotes the direction cosine vector from the receiver to the ith satellite, and it can be calculated by the approximate position of the receiver and the position of the ith satellite in three dimensions. Besides, it satisfies that $\|\boldsymbol{h}_i\| = 1$. That is, the tips of these vectors lie on the surface of a unit sphere.

Similarly, if one additional constellation (i.e., BDS, GLOLASS, and Galileo) is combined with GPS, the corresponding measurement equation is expressed as

$$z_2 = \boldsymbol{H}_2 \Delta\boldsymbol{x}_2 = \boldsymbol{H}_2 [\Delta\boldsymbol{r} \quad c\Delta t_2]^T \tag{3}$$

with $\boldsymbol{H}_2 \in R^{\beta\times 4}$. The number of row ($\beta$) equals the number of tracked satellites in the additional single constellation. The parameter $c\Delta t_2$ is the receiver clock bias related with the corresponding constellation. The receiver clock bias in Eq. (3) and that in Eq. (1) are different (Wang et al., 2011; Guo et al., 2015).

Combining Eq. (3) with Eq. (1) leads to

$$z = \boldsymbol{H}\Delta\boldsymbol{x} \tag{4}$$

where

$$\begin{cases} z = [\, z_1^T \quad z_2^T \,]^T \\ \Delta\boldsymbol{x} = [\, \Delta\boldsymbol{r} \quad c\Delta t_1 \quad c\Delta t_2 \,]^T \end{cases} \tag{5}$$

According to the linear measurement equation in Eq. (4), if we assume that the measurements from different satellites have the same accuracy and they are also statistically independent, then the GDOP in the single-point positioning for dual-GNSS constellations can be defined as

$$DGDOP = \sqrt{tr[(\boldsymbol{H}^T\boldsymbol{H})^{-1}]} \tag{6}$$

where the design matrix is given by

$$\boldsymbol{H} = \begin{bmatrix} \boldsymbol{h}_1^T & \cdots & \boldsymbol{h}_\alpha^T & \boldsymbol{h}_{\alpha+1}^T & \cdots & \boldsymbol{h}_{\alpha+\beta}^T \\ 1 & \cdots & 1 & 0 & \cdots & 0 \\ 0 & \cdots & 0 & 1 & \cdots & 1 \end{bmatrix}^T \tag{7}$$