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The impact of the orbital decay of the LAGEOS satellites on the frame-dragging tests

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Abstract

The laser-tracked geodetic satellites LAGEOS, LAGEOS II and LARES are currently employed, among other things, to measure the general relativistic Lense–Thirring effect in the gravitomagnetic field of the spinning Earth with the hope of providing a more accurate test of such a prediction of the Einstein's theory of gravitation than the existing ones. The secular decay $\dot{a}$ of the semimajor axes a of such spacecrafts, recently measured in an independent way to a $\sigma_{\dot{a}} \approx 0.1$–$0.01$ m yr^{-1} accuracy level, may indirectly impact the proposed relativistic experiment through its connection with the classical orbital precessions induced by the Earth's oblateness J_2. Indeed, the systematic bias due to the current measurement errors $\sigma_{\dot{a}}$ is of the same order of magnitude of, or even larger than, the expected relativistic signal itself; moreover, it grows linearly with the time span T of the analysis. Therefore, the parameter-fitting algorithms must be properly updated in order to suitably cope with such a new source of systematic uncertainty. Otherwise, an improvement of one-two orders of magnitude in measuring the orbital decay of the satellites of the LAGEOS family would be required to reduce this source of systematic uncertainty to a percent fraction of the Lense–Thirring signature.

Keywords: Experimental studies of gravity; Experimental tests of gravitational theories; Satellite orbits; Harmonics of the gravity potential field

1. Introduction

The so-called general relativistic Lense–Thirring effect (Lense and Thirring, 1918) consists of small secular orbital precessions affecting the motion of a test particle in geodesic motion about a central rotating body. They are driven by the gravitomagnetic[1] component of the gravitational field of the spinning mass due its proper angular momentum. As far as the Earth is concerned, the Satellite Laser Ranging (SLR) technique has been used so far in conjunction with the geodetic satellites LAGEOS and LAGEOS II to attempt to measure such a manifestation of frame-dragging[2] (Ciufolini and Pavlis, 2004; Ciufolini et al., 2010). A third member of this class of passive, spherical laser targets, named LARES, was launched a few years ago (Paolozzi and Ciufolini, 2013), and it will be used to try to refine the existing tests in view of its improved manufacturing and design which will reduce the direct impact of the non-gravitational perturbations; according to its proponents, it should be possible to reach a $\approx 1\%$ total accuracy (Ciufolini et al., 2013). For recent overviews of the performed and ongoing attempts to detect the Lense–Thirring effect with artificial Earth's satellites and related discussions on the realistic accuracy level reached, see,

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[1] Such a denomination comes from the purely formal resemblance of the linearized Einstein field equations in the weak-field and slow-motion approximation with the Maxwell equations of electromagnetism.

[2] The GP-B mission (Everitt et al., 2011) measured another gravitomagnetic effect in the field of the Earth.

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e.g., Iorio et al. (2011), Ciufolini et al. (2012), Renzetti (2013), Iorio et al. (2013), Ciufolini et al. (2013), and Renzetti (2015) and references therein.

In this paper, we want to address a further source of potential systematic uncertainty in the total error budget of the ongoing relativity experiment involving the three satellites of the LAGEOS family. Basically, the Newtonian precession of the orbital plane induced by the asphericity of the Earth (Xu, 2008) is nominally several orders of magnitude larger than the general relativistic gravitomagnetic one. Such a classical feature, which acts as a major systematic error, depends, among other factors, on some of the satellite's orbital parameters; thus, long-term variations in them indirectly affect also the orbital precession itself inducing a further bias which has to be properly assessed and subtracted from the signal from which the Lense–Thirring effect should be extracted. In particular, it turns out that the size of the orbits of the satellites considered is steadily diminishing over the years (Sośnica et al., 2014; Sośnica, 2014). Although the gravitomagnetic field does not directly affect such an orbital feature, the resulting additional shift in the orbital plane may alias the corresponding Lense–Thirring precession by potentially corrupting the overall accuracy level in this frame-dragging test in absence of appropriate countermeasures in the data reduction procedure.

The relevant orbital parameters of LAGEOS, LAGEOS II, LARES are summarized in Table 1, in which the following values of the key fundamental constants and geophysical parameters of the Earth were adopted: $G = 66,451.1\ \mathrm{kg^{-1}\ m^3\ yr^{-2}}$, $c = 9.46073 \times 10^{15}\ \mathrm{m\ yr^{-1}}$, $GM = 3.96959 \times 10^{29}\ \mathrm{m^3\ yr^{-2}}$, $S = 1.84928 \times 10^{41}\ \mathrm{kg\ m^2\ yr^{-2}}$, $J_2 = 0.00108262$, $R = 6{,}378{,}136.6$ m.

2. Evaluating the bias due to the satellite's orbital decay

2.1. The classical and relativistic orbital precessions

In the following, a coordinate system whose z axis is aligned with the Earth's spin axis will be assumed.

For a given value of its inclination I to the Earth's equator, the location of the satellite's orbital plane in the inertial space is determined by the longitude of the ascending node Ω: it is in angle in the equatorial plane reckoned from a reference x direction to the line of the nodes which, in turn, is determined by the intersection of the orbital plane with the equatorial plane itself. If the primary was pointlike or perfectly spherical, the orbital plane would remain fixed in space, so that Ω would keep a constant value. Actually, every astronomical body such as our planet does rotate, so that its shape is distorted by the centrifugal effects which make its gravitational field to depart from spherical symmetry. To the Newtonian level, the gravitational potential is usually expanded in multipolar coefficients; among them, the even zonal ones $J_\ell, \ell = 2, 4, 6, \ldots$ are of particular importance since they induce orbital perturbations which do not vanish when averaged out over one orbital revolution. In particular, the satellite's node rate induced by the first even zonal J_2 of the central body is Xu (2008)

$$\dot{\Omega}_{J_2} = -\frac{3}{2} n \left(\frac{R}{a}\right)^2 \frac{\cos I J_2}{\left(1 - e^2\right)^2}, \tag{1}$$

where the Keplerian mean motion is

$$n = \sqrt{\frac{GM}{a^3}}. \tag{2}$$

In Eqs. (1) and (2), G is the Newtonian constant of gravitation, and M, R, J_2 are the mass, the equatorial radius and the dimensionless first even zonal harmonic of the primary, respectively; a and e are the satellite's semimajor axis and eccentricity determining its orbital size and shape, respectively.

The Lense–Thirring node rate is Lense and Thirring (1918)

$$\dot{\Omega}_{\mathrm{LT}} = \frac{2GS}{c^2 a^3 \left(1 - e^2\right)^{3/2}}, \tag{3}$$

where c is the speed of light in vacuum and S is the proper angular momentum of the primary.

For the LAGEOS-type satellites, the Newtonian disturbing precessions due to J_2 and the other even zonals of higher degree are nominally several orders of magnitude larger than the relativistic ones; thus, although the geopotential is modeled in the softwares routinely used to analyze satellites' data, their mismodeled components due to the current uncertainties $\delta J_\ell, \ell = 2, 4, 6, \ldots$ are still too large with respect to the Lense–Thirring precession of Eq. (3). As a consequence, several strategies have been devised so far to circumvent such a potentially fatal drawback; for details, see, e.g., Iorio et al. (2011), Renzetti (2013), and Ciufolini et al. (2013) and references therein.

As we will show in the next Section, the present-day level of uncertainty in our knowledge of the even zonals is not the only source of systematic bias connected with the classical node precession of Eq. (1).

Table 1
Relevant orbital parameters of the satellites of the LAGEOS family: a is the semimajor axis, e is the eccentricity, I is the inclination to the Earth's equator, n is the Keplerian mean motion.

	LAGEOS	LAGEOS II	LARES
a (km)	12,274	12,158	7,820
e	0.0039	0.0137	0.0007
I (deg)	109.90	52.67	69.50
n ($\mathrm{yr^{-1}}$)	1.465×10^4	1.486×10^4	2.881×10^4

2.2. When the semimajor axis does secularly vary

By allowing for a secular variation of the semimajor axis a

$$a(t) = a_0 + \dot{a}t, \tag{4}$$