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Numerical orbit integration based on Lie series with use of parallel computing techniques

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Abstract

This article outlines necessary steps to perform numerical orbit integrations based on a Lie series approach. Its implementation requires an efficient evaluation of resulting series coefficients. As an example we treat the classical main problem in satellite orbit calculation (J_2 only) and the case of a 4×4-gravity field. All calculations were performed in very high precision with up to 100 significant digits. In comparison to independent third party computations this approach led to superior results referring to the verifiable constancy of various integrals of motion. To achieve a performance similar to classical numerical integrations in terms of acceptable computing time, at least for non-Keplerian motion problems, we exploited parallel computing capabilities. For our examples, run times were improved by several orders of magnitude, depending on the actual chosen precision level (up to a factor of 50,000 in case of double precision). Here we present the mathematical framework of the proposed orbital integration scheme as well as the work flow for its application in a multi-core, parallel computing environment.

Keywords: Numerical integration; Satellite orbits; Lie series; Parallel computing; OpenMP

1. Introduction

In physical geodesy, one of the main tasks is to establish and monitor a global geodetic observing system (GGOS). Such a GGOS (Rummel et al., 2002) comprises three major parts, namely the geometry, rotation, and gravity field of the Earth. Today's high demands on the accuracy level in the detection of state changes requires the exploitation of precise satellite based measurement techniques. Satellite mission planning requires preliminary studies based on different scenarios. In practice, various effects are superimposed and a successful interpretation in the postprocessing will be based on the capability to disentangle individual contributions. To achieve this goal, sufficiently long time series of data and its spectral analysis is required.

The consideration of physical effects comprises the simulation of its observability by a given instrumentation, experimental design, and the actual set up of measurement devices. Those devices are either ground-based surveying instruments (e.g. gravimeters) or space-based sensor-carrying platforms (e.g. gradiometers, accelerometers) that are mounted aboard Earth orbiting satellites like GRACE (Tapley et al., 2004), GOCE (Rummel and Gruber, 2010). In general, observations are made in the time domain, whereas the postprocessing or analysis of residuals is performed in the spectral domain in order to improve the modeling of involved physical processes, based on determined amplitudes, frequencies and phases.

As an example, changes in the gravitational field of the Earth due to (interior and/or exterior) mass variations result in a detectable perturbation of satellite orbits (e.g. Rummel et al., 2011). Vice versa, precise knowledge of satellite orbits is exploited to solve the inverse problem, i.e., detection of changes in the Earth's system. Perturbations

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relate to an a priori model of the satellite's trajectory. Studying residuals means to compare observed satellite states (position and velocity) against calculated model-dependent states. The former are obtainable by various satellite geodetic techniques, e.g., GNSS positioning (Dow et al., 2009), satellite laser ranging (Pearlman et al., 2002), satellite-to-satellite tracking, DORIS measurements (Willis et al., 2010), whereas the calculated states are given by the solution of corresponding equations of motion.

In order to perform the above-mentioned preliminary studies with the help of simulations one needs the ability to reliably create comparatively long accurate orbital arcs. Orbital integration either makes use of classical numerical integration techniques, analytical orbital theories (e.g. Cui, 1997) or a combination of both, so-called semi-analytical approaches (e.g. Dallas, 1970).

Whilst classical numerical integrators are easy to implement and fast running, their results rapidly deteriorate with time due to the accumulation of numerical errors. Furthermore, they operate in the time domain and thus only yield time series that subsequently have to be analyzed in the spectral domain by suitable techniques. However, any results obtained in this way will only be valid for the very special case under consideration and thus do not allow one to draw universally valid conclusions. In other words, one can never be sure to find the same or a similar evolution of another satellite orbit due to a certain physical effect just by looking at a single simulation and study of residual data. Instead, one had to perform a whole set of slightly varying integrations in order to get at least an approximative understanding of the actual chain of causation or qualitative behavior of the dynamical system.

On the other hand, existing analytical theories are comparatively complex (Mai et al., 2008) and not as easy to implement as numerical schemes. Nonetheless, their advantage is to work within the spectral domain right from the beginning and thus they allow for a much simpler assignment of physical causes to observed orbital effects. Once such a theory is implemented it is faster than a numerical approach in terms of computation time, especially for very long orbital arcs. The main idea here is to use a remove-restore approach. An intermediate orbit is obtained from the true orbit via a reduction procedure. Then this intermediate orbit can be propagated by closed formulas to any desired epoch in time without any intermediary steps, thus avoiding the accumulation of numerical errors as in a usual stepwise (numerical) integration. Afterwards one corrects for the difference between the intermediary and true orbit to find the actual state at the requested epoch. Details on such a procedure, based on a Lie series approach, and persisting limitations can be found in Mai (2005).

In an attempt to combine the advantages of both methods, analytical and numerical integration techniques, we propose a numerical integration scheme based on Lie series (Mai, 2011). This approach, although being comparatively easy in mathematical formulation (Section 2), is challenging in implementation. The efficient evaluation of the resulting Lie series coefficients required the use of parallel programming techniques (Geyer, 2012) (Section 3).

Remark: in case of pure point mass interactions (Keplerian motion), especially for large N-body problems, Lie series approaches show comparatively high performance even without parallelization (Eggl and Dvorak, 2010).

Still being based on polynomials in time, the proposed integration scheme could be regarded as kind of a usual Taylor integrator. At this stage, in our algorithm, we have not yet implemented recurrence relations, which are fundamental in well-established recurrence power series methods (Broucke, 1971). The latter are implemented in a series of more general software packages (e.g. Abad et al., 2012), several of which have been subject to comparative studies (Gerlach, 2011). Our own comparisons relate to a series of specific satellite orbit integrators.

We present an outlook for remaining steps in order to derive an integration procedure that is competitive to classical numerical integration techniques, which may result in a semi-analytical orbital theory.

2. Lie series approach

Lie series (Gröbner, 1960) can be used to construct analytical orbital theories and numerical integration schemes alike. Basic ideas for its application in the solution of equations of motion (EOM) of simple artificial motion problems are presented in Mai (2008).

2.1. Problem statement

In the following we will focus on a certain non-idealized two-body problem, namely the relative motion of an artificial satellite (secondary body treated as a point mass of negligible mass) about the Earth (primary body being a rigid and uniformly rotating non-point mass of mass $M_\oplus$). The Earth is characterized by its gravitational parameter $\mu_\oplus$, mean equatorial radius $a_\oplus$, and proper rotation $\omega_\oplus$, here we choose the nominal values

$$\mu_\oplus = GM_\oplus = 398600.4415\ \mathrm{km}^3/\mathrm{s}^2,$$
$$a_\oplus = 6378.1363\ \mathrm{km},$$
$$\omega_\oplus = 2\pi/86164\ \mathrm{s}.$$

Those numbers are consistent with the chosen Joint Gravity Model (version 3) JGM-3 (Tapley et al., 1996). Any further parameters, e.g., spherical harmonic coefficients of the series expansion for Earth's gravitational potential, will also relate to this model.

Given the initial state $\mathbf{z}(t_0) = \mathbf{z}_0 = (\mathbf{r}_0, \mathbf{v}_0)$ of a satellite, where $\mathbf{r}$ and $\mathbf{v}$ are its position and velocity in a quasi-inertial Earth-fixed coordinate system, we want to calculate the satellite's orbital evolution under the gravitational attraction solely due to Earth as described via the gradient of its potential $V_\oplus(\mathbf{r}, t)$ such that the following (relative) EOM has to be solved:

$$\ddot{\mathbf{r}} + \nabla_{\mathbf{r}} V_\oplus(\mathbf{r}, t) = \mathbf{0}. \quad (1)$$