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DECAY OF SATELLITE ORBITS USING K-S ELEMENTS IN AN OBLATE DIURNALLY VARYING ATMOSPHERE WITH SCALE HEIGHT DEPENDENT ON ALTITUDE

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ABSTRACT

A non-singular analytical theory for the motion of near-Earth satellite orbits with the air drag effect is developed in terms of the K-S elements utilizing an analytical, oblate diurnally varying atmospheric model with varying scale height, dependent on altitude. The series expansions include up to fourth-order terms in eccentricity (e) and c, a small parameter dependent on the flattening of the atmosphere. Only two of the nine equations are solved analytically to compute the state vector at the end of each revolution due to symmetry in the K-S element equations. Numerical studies are done over a wide range of orbital parameters. A numerical comparison with numerically integrated values of the change in the orbital parameters: semi-major axis and eccentricity is made with the extended theory up to fourth-order terms of Swinerd and Boulton. It is observed that the theory in terms of the K-S elements provides better accuracy than the extended theory of Swinerd and Boulton, when compared with the numerically integrated values.

INTRODUCTION

It is well known that the Earth's atmosphere is non-stationary and does not possess spherical symmetry and varies, in general, considerably during the lifetime of a satellite. The change in the distribution of atmospheric density with time is primarily caused by the variability of the ultraviolet and corpuscular fluxes from the Sun. As these fluxes change relatively slowly with time, the atmosphere can be considered stationary at time intervals comparable to the satellite orbital period around the Earth. The non-spherical nature of the atmosphere is primarily due to Earth's gravitational field and the heating of the illuminated part of the Earth's atmosphere by solar radiation. The atmospheric density at a given distance from the center of the Earth will depend upon the geographic latitude and longitude, and on the Sun's location in the celestial sphere. Heating of the illuminated part of the Earth's atmosphere by the Sun results in a deformation in the upper atmosphere and an acquired pear shape directed in an acute ring approximately towards the Sun. Thus the selection of an appropriate atmospheric model has great significance in the study of the perturbations of satellite motion caused by air drag.

In order to account for the drag perturbations, it is necessary to accurately model the density of the Earth's atmosphere. Some of the important atmospheric models used frequently in the literature are CIRA (1972), Jacchia (1977), Barlier et al. (1978), MSIS-86 (Hedin, 1987) and MSIS-90 (Hedin, 1991). MSIS-90 uses analytic models to model the lower altitudes to account for disturbances such as solar activity, magnetic storms, and daily variations, as well as latitude, longitude, and monthly variations.

Analytical solutions of artificial satellite motion in an atmosphere, which considers the non-sphericity of the distribution of the atmospheric density in which diurnal and atmospheric oblateness effects are treated in combination

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were carried out by Santora (1975), Swinerd and Boulton (1982) and a few others. Swinerd-Boulton's theory is up to third-order terms in e and c. In both, Swinerd-Boulton and Boulton (1983) theories, the variation of density scale height with altitude was included. Sharma (1997) developed an analytical solution in terms of the K-S elements up to third-order terms in e and c, by including the effects of the atmospheric oblateness and diurnal bulge. However, the density scale height was assumed to be constant.

In this paper, we extend Sharma's theory up to fourth-order terms in e and c and include the variation of the density scale-height with altitude. Only two of the nine equations need to be solved analytically to compute the state vector at the end of each revolution due to symmetry in the K-S element equations. Swinerd and Boulton, and Boulton solutions have also been extended up to fourth-order terms in e and c for comparison purpose. These solutions are compared numerically with the present solution obtained in terms of the K-S elements. Numerical comparisons are made with four test cases having perigee altitudes of 175, 240, 300 and 400 km, with each case having low and high inclination of 5^0 and 80^0 and with a high value of 0.2 for the rate of increase of the density scale height H with the distance r. At each perigee height, two values of e are chosen as 0.1 and 0.2. The series expansions and simplifications are done using the software MATHEMATICA (Wolfram, 1996), available at Vikram Sarabhai Space Centre, Thiruvananthapuram. As our aim is to compare the numerical and analytical solutions, we have utilized a relatively simple, Jacchia (1977) atmospheric model to compute the density and density scale height values. Two distinct advantages of the present solution are seen over the theory of Swinerd and Boulton, and Boulton. In the present theory only two equations are handled analytically against three by Swinerd and Boulton, and Boulton. The present solution is non-singular, whereas in Boulton's expression of argument of perigee variation ($\Delta\omega$), e is in the denominator.

EQUATIONS OF MOTION

The K-S element equations of motion of a satellite under the effect of additional perturbing force $\vec{P}$ are (Stiefel and Scheifele, 1971)

$$\frac{dw}{dE}=-\frac{1}{2w}(\vec{u}^*,L^T\vec{P}), \tag{1}$$

$$\frac{d\tau}{dE}=\frac{1}{8w^3}[K^2+r(\vec{u},L^T\vec{P})-16w\frac{dw}{dE}(\vec{u},\vec{u}^*)], \tag{2}$$

$$\frac{d\vec{\alpha}}{dE}=\vec{Q}\sin(\frac{E}{2}),\quad \frac{d\vec{\beta}}{dE}=-\vec{Q}\cos(\frac{E}{2}),\quad \vec{Q}=\frac{1}{4w^2}[-rL^T(\vec{u})\vec{P}+8w\frac{dw}{dE}\vec{u}^*], \tag{3}$$

$$\vec{u}=(u_1,u_2,u_3,u_4)=\vec{\alpha}\cos(\frac{E}{2})+\vec{\beta}\sin(\frac{E}{2}),\quad \vec{u}^*=\frac{d\vec{u}}{dE}=\frac{1}{2}[-\vec{\alpha}\sin(\frac{E}{2})+\vec{\beta}\cos(\frac{E}{2})],$$

$$\tau=t+\frac{1}{w}(\vec{u},\vec{u}^*),\qquad \vec{x}=(x_1,x_2,x_3)=L(\vec{u})\vec{u},\ r=(x_1^2+x_2^2+x_3^2)^{\frac{1}{2}}=u_1^2+u_2^2+u_3^2+u_4^2,$$

$$\alpha_4\beta_1-\alpha_3\beta_2+\alpha_2\beta_3-\alpha_1\beta_4=0, \tag{4}$$

$$\text{and}\quad L(\vec{u})=\begin{bmatrix} u_1 & -u_2 & -u_3 & u_4 \\ u_2 & u_1 & -u_4 & -u_3 \\ u_3 & u_4 & u_1 & u_2 \\ u_4 & -u_3 & u_2 & -u_1 \end{bmatrix}, \tag{5}$$

where Eq. (4), is the bilinear relation and E, w, t, r and K^2 are, respectively, the eccentric anomaly, angular frequency, physical time, radial distance and the gravitational constant. Knowing the position and velocity vector $\vec{x}$ and $\dot{\vec{x}}$ at the instant t = 0, the values of r, w, τ, u_i and u_i* can be computed and by adopting E = 0 as the initial value of the eccentric anomaly, we obtain

$$\vec{\alpha}=\vec{u},\vec{\beta}=2\vec{u}^*$$

If $\vec{P}$ is the aerodynamic drag force per unit mass on a satellite of mass m, (King-Hele, D.G, 1987)

$$\vec{P}=-\frac{1}{2}\rho\delta|\vec{v}|\vec{v},\quad \delta=FAC_D/m,\qquad F=[1-r_{p0}\Lambda\cos i_0/v_{p0}]^2, \tag{6}$$