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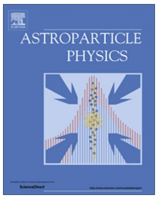

Analyzing the data from X-ray polarimeters with Stokes parameters

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ABSTRACT

X-ray polarimetry promises to deliver unique information about the geometry of the inner accretion flow of astrophysical black holes and the nature of matter and electromagnetism in and around neutron stars. In this paper, we discuss the possibility to use Stokes parameters – a commonly used tool in radio, infrared, and optical polarimetry – to analyze the data from X-ray polarimeters such as scattering polarimeters and photoelectric effect polarimeters, which measure the linear polarization of the detected X-rays. Based on the azimuthal scattering angle (in the case of a scattering polarimeter) or the azimuthal component of the angle of the electron ejection (in the case of a photoelectric effect polarimeter), the Stokes parameters can be calculated for each event recorded in the detector. Owing to the additive nature of Stokes parameters, the analysis reduces to adding the Stokes parameters of the individual events and subtracting the Stokes parameters characterizing the background (if present). The main strength of this kind of analysis is that the errors on the Stokes parameters can be computed easily and are well behaved – in stark contrast of the errors on the polarization fraction and polarization direction. We demonstrate the power of the Stokes analysis by deriving several useful formulae, e.g. the expected error on the polarization fraction and polarization direction for a detection of NS signal and NBG background events, the optimal observation times of the signal and background regions in the presence of non-negligible background contamination of the signal, and the minimum detectable polarization (MDP) that can be achieved when following this prescription.

1. Introduction

The measurement of the linear polarization of the X-rays from cosmic sources holds the promise to provide geometrical information about the innermost regions of the most extreme objects in the universe, black holes and neutron stars [1–3]. These systems emit copious amounts of X-rays but are too small to be imaged with current technology. Despite the scientific potential of X-ray polarimetry, only one dedicated satellite-borne X-ray polarimeter has been flown so far. The Bragg polarimeter on board the OSO-8 satellite launched in 1978 measured a polarization fraction of the Crab Nebula of about 20% at energies of 2.6 and 5.2 keV [4]. Since then, three more X-ray polarization measurements have been published: In 2008, the instruments SPI and IBIS on board the INTEGRAL satellite reported polarization fractions of the Crab Nebula of 46 ± 10% [5] and >72% [6], respectively, with the polarization direction aligned with the X-ray jet. For the stellar mass black hole Cygnus X-1 in an X-ray binary, a polarization fraction of $40 \pm 10\%$ in the 230 to 400 keV range and $> 75\%$ [7] and $67 \pm 30\%$ [8] at higher energies have been reported. For a number of Gamma-Ray Bursts, tentative evidence for polarized emission has been published [9–11], but the measurements are plagued by large statistical and systematic uncertainties.

More recently, various wider-bandpass polarimeters have been developed, including photoelectric effect polarimeters (e.g. the polarimeters of the proposed GEMS [12] and XIPE [13] missions) and scattering polarimeters (e.g. the polarimeters of the X-Calibur [14] and PoGOLite [15] missions). Photoelectric effect polarimeters track the direction of photoelectrons which are preferentially emitted parallel to the electric field of the incoming photons (e.g. Ref. [16]). Scattering polarimeters measure the direction into which the photons scatter and make use of the fact that photons scatter preferentially perpendicular to the electric field direction of the X-ray beam (e.g. Ref. [17]). Unlike radio or optical telescopes, which measure the intensity of the radiation from the source, most X-ray telescopes detect individual photons. The linear polarization of the X-rays leads to a sinusoidal modulation of the azimuth distribution of events with a 180° period and a phase depending on the polarization direction. The relative amplitude

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of the modulation corresponds to the polarization fraction. The standard method for determining the linear polarization fraction and angle of an X-ray beam is to fit a sine function to the observed azimuth distribution.

In 1852, George Gabriel Stokes introduced a set of four parameters which are sufficient to completely describe the polarization properties of a quasi-monochromatic beam with arbitrary linear and circular polarization properties [18]. These four *Stokes parameters* are linear, i.e. the intensity and polarization of a superposition of light beams from two different sources is described by the sum of their Stokes parameters. Radio antennas and optical telescopes equipped with polarization filters – being sensitive to certain polarization directions – can basically measure Stokes Parameters directly (e.g. Ref. [19]). Owing to their additive properties, Stokes parameters are also used in theoretical calculations, e.g. in radiative transfer calculations [20] and in quantum mechanical calculations involving polarized photons [21,22].

In this paper, we discuss the use of Stokes parameters in the analysis of the data from X-ray polarimeters. We define the Stokes parameters for an idealized polarimeter with uniform acceptance and discuss their statistical properties in Section 2. In Section 3 we describe the implications for the analysis of X-ray polarimetry data. We give a detailed discussion of how to calculate errors on the polarization fraction and polarization direction in Section 4. In Section 5, we use the results from the previous sections to optimize the observation strategy in the presence of non-negligible backgrounds. Finally, in Section 6 we summarize our findings. In Appendix A, we give the equations in modified form for the case of a polarimeter with non-uniform detector acceptance.

2. The Stokes parameters and their statistical properties

For a classical electromagnetic “quasi-monochromatic wave” (a wave which is 100% polarized over short time intervals comparable to the period of the wave, but whose polarization properties change on longer time scales) the Stokes parameters can be defined by time averages (denoted by “$\langle\,\rangle$”) of the electric field strength along two orthogonal directions (see e.g. Refs. [20,23]). Assuming a wave propagating along the z-axis towards larger z, the defining equations read:

$$S_0 = I = \langle E_x^2 + E_y^2 \rangle, \tag{1a}$$

$$S_1 = Q = \langle E_x^2 - E_y^2 \rangle, \tag{1b}$$

$$S_2 = U = \langle 2E_xE_y \cos\delta \rangle, \tag{1c}$$

$$S_3 = V = \langle 2E_xE_y \sin\delta \rangle. \tag{1d}$$

Here, E_x (E_y) is proportional to the instantaneous electric field along the x-axis (y-axis), and δ is the lag of E_y behind E_x. All four Stokes parameters have units of intensity (or flux). The parameter I is the intensity (or flux) of the wave, Q and U depend on the linear polarization properties, and V on the circular polarization properties. The parameter Q equals I ($-I$) for a 100% linearly polarized wave with an **E**-field vector along the x-axis (y-axis); U equals I ($-I$) for a 100% linearly polarized wave with an **E**-field vector along the diagonal between the x-axis and the y-axis (the negative x-axis and the y-axis); V equals I ($-I$) for 100% circularly right handed (left handed) polarized light. Fig. 1 illustrates this. The appropriate reference coordinate system for Q and U has been defined by the IAU [24]: $+Q$ corresponds to a linear polarization in North/South direction, $-Q$ to a polarization in East/West direction, and $+U$ corresponds to a polarization along the North East/South West diagonal.

For a monochromatic (100% polarized wave) the identity

$$I = \sqrt{Q^2 + U^2 + V^2} \tag{2}$$

holds. For a linearly polarized wave ($V = 0$), the equation simplifies to $I = \sqrt{Q^2 + U^2}$, and Q and U are given by the polarization direction ψ (the angle between the x-axis and the electric field direction) by:

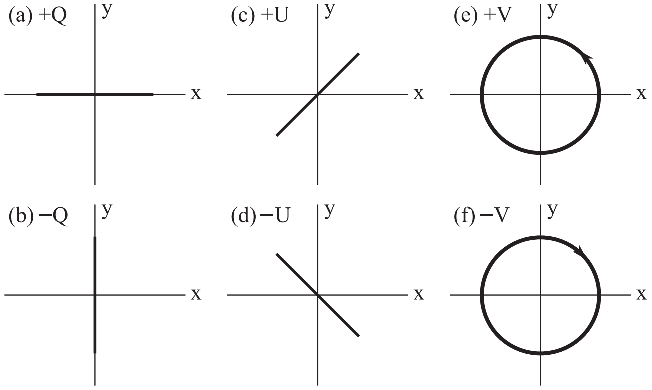

Fig. 1. Polarization for different values of the Stokes parameters. (a) $Q > 0,\ U = 0,\ V = 0$; (b) $Q < 0,\ U = 0,\ V = 0$; (c) $U > 0,\ Q = 0,\ V = 0$; (d) $U < 0,\ Q = 0,\ V = 0$; (e) $V > 0,\ Q = 0,\ U = 0$; (f) $V < 0,\ Q = 0,\ U = 0$.

$$Q = \cos 2\psi, \tag{3a}$$

$$U = \sin 2\psi, \tag{3b}$$

which implies:

$$\tan 2\psi = \frac{U}{Q}. \tag{4}$$

It can be shown that the Stokes parameters of an ensemble of quasi-monochromatic waves are additive: the Stokes parameters of the superposition of the waves equal the sum of the Stokes parameters of the individual waves. Such an ensemble of waves can be described as a superposition of unpolarized and polarized waves. It can be shown that the polarization fraction (the intensity or flux of the polarized waves) is then given by:

$$p = \frac{\sqrt{Q^2 + U^2 + V^2}}{I}. \tag{5}$$

In the case of linearly polarized waves ($V = 0$), the linear polarization fraction is given by

$$p_l = \frac{\sqrt{Q^2 + U^2}}{I} \tag{6}$$

and the polarization direction (the direction of the electric field vector of the linearly polarized waves) can be inferred from the equation:

$$\tan 2\psi = \frac{U}{Q}. \tag{7}$$

We now discuss how to use the Stokes parameters for the analysis of the data from an X-ray polarimeter, i.e. a scattering polarimeter or a photoelectric effect polarimeter. For both types of polarimeters, a data set consists of a list of K angles $\{\varphi_k\}$ with $k = 1 \ldots K$, which are related to the most likely azimuthal angle ψ_k of the electric field vector. In case of photoelectric effect polarimeters, $\psi_k = \varphi_k$, whereas in case of scattering polarimeters $\psi_k = \varphi_k - 90^\circ$. These angles exhibit a sinusoidal modulation with period 180° [1–3]:

$$f(\psi) = \frac{1}{2\pi}(1 + p_0\,\mu\cos(2(\psi - \psi_0))) \tag{8}$$

with p_0 being the true polarization fraction, ψ_0 giving the expected direction where the ψ-distribution peaks, and μ being the