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Research paper

Mathematical links between optimum solar collector tilts in isotropic sky for intercepting maximum solar irradiance

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ABSTRACT

The paper presents a mathematical modeling of the optimum tilt for solar collectors for intercepting maximum solar irradiance (power density), at different geographical locations, periods of time and different base-ground types. The solar irradiance received by the collector is estimated based on isotropic sky analysis models, namely Hottel & Woertz model and Liu & Jordan model. The optimum value for the tilt is considered for maximum hourly and respectively daily noon incident solar irradiance. This paper emphasizes the mathematical link between the optima computed under the two considered models assumptions. Also the ground reflectance factor influence on the optimum tilt difference between considered models is presented related to latitude.

1. Introduction

Solar collectors are widely used for different applications, from ordinary water heating during summer in household use (achieved through non-concentrating collectors) to solar power systems (using concentrating collectors). For both types of solar collectors, the target is to catch either maximum possible solar irradiance or solar energy on the aperture area of the collector. Non-concentrating collectors are characterized by the same area for intercepting and absorbing solar radiation, equivalent to unity concentration ratio. Contrary, concentrating collectors intercept solar radiation by a larger aperture area in comparison to the absorbed (receiver) one, thus concentration ratios are higher than unity.

In the first case, the collectors are usually fixed mounted all along a day, often tilted on North–South direction, facing southwards in the Northern hemisphere. Maximum intercepted solar energy is targeted, for which optimum tilt is then determined. They could be daily, monthly or seasonally adjusted or even fixed over the entire year of operation at a specified tilt.

In the second case, Sun tracking mechanisms are usually used for moving and tilting the concentrating collector so that the Sun rays fall perpendicular on the aperture area. In this regards, there are two kinds of tracking mechanisms: single-axis and two-axis one. For example, parabolic trough collectors, characterized by 10 to 40 concentration ratios (Kalogirou, 2004), are usually designed with a single-axis tracking mechanism, moving the collector either on East–West axis, or North–South one. For parabolic dish reflectors having concentration ratios higher than 100, two-axes tracking mechanisms are usually applied, so that the Sun rays are intercepted perpendicular to the aperture area all day long (Kalogirou, 2004). Their concentration ratios could arrive to more than 10000. Feidt et al. (2004) presented in a published work elements regarding optimum concentration ratios for different kind of parabolic solar concentrators.

As a conclusion, different optimizing criteria might be sought when computing optimum tilt or path for the solar collector, mainly depending on the collector type and economic reasons.

As non-concentrating collectors are intercepting both beam and diffuse components of solar radiation, maximizing received total solar energy is a common criterion used for determining the optimum tilt. In case of collectors having concentration ratios higher than 10, only beam component of the solar radiation is mainly used (Prapas et al., 1987) (as cited by Kalogirou (2004)) and thus targeted.

In technical literature one may find different proposed methods for determining the optimum tilt, considering different criteria and expressing the tilt function on latitude or declination or other local constants. Maximizing total incident solar energy on the collector, either in atmospheric conditions or extraterrestrial ones,

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Nomenclature

a_0, a_1	constants for standard atmosphere
C	notation for term given by Eq. (28)
G	solar irradiance, Wm^{-2}
k	constant for standard atmosphere
n	number of a day in an year

Greek symbols

α	solar absorptance
β	slope of a tilted surface, deg
δ	solar declination
φ	geographical latitude, deg
γ	surface azimuth angle, deg
ρ	diffuse reflectance
τ	atmosphere transmittance
θ	angle of incidence of beam radiation on a surface, deg
θ_z	zenith angle, deg
ω	hour angle, deg

Subscripts and superscripts

B	beam
CS	clear sky conditions
D	diffuse
ET	extraterrestrial radiation
FPC	flat plate collectors
g	ground
(HW)	Hottel and Woertz model
(LJ)	Liu and Jordan model
max	maximum value
n	normal to a plane
(N)	North hemisphere
opt	optimum
(S)	South hemisphere
SC	solar constant
t	tilt
T	total
12 h	at 12 O'clock

is the main applied criterion. A brief summary of the published papers regarding flat plate collectors is presented by (Armstrong and Hurley, 2010) who deduced the optimum tilt particularly for climates susceptible to frequently overcast skies and validated for all other climate types. Some of the published methods calculate the optimum tilt by maximizing the extraterrestrial solar energy (Soulayman, 1991; Soulayman and Sabbagh, 2014; Gunerhan, 2005), others the direct solar radiation incident on the collector (Kern and Harris, 1975) and others maximize the total incident solar energy (Tamimi, 2011; Agarwal et al., 2012; Bakirci, 2012; Morcos, 1994). Particular correlations for optimum tilt have been derived at different latitudes.

Based on the firstly derived isotropic sky model, Hottel and Woertz model, Tamimi (2011) proved a mathematical dependence of the optimum tilt on latitude, declination and length of the day and applied it for Amman, Jordan. The optimization criterion was the daily terrestrial solar energy, expressed as the product between average clearness index (constant value) and extraterrestrial solar energy. An optimum value is provided for each month and finally a yearly average tilt was proposed, correlated to NASA measurements. The proposed final correlation is $\beta_{opt}=\varphi\pm(10°-15°)$.

Agarwal et al. (2012) also maximized terrestrial solar energy for India and compared the tilts numerically derived by 4 isotropic and 4 anisotropic models. Numerical and graphical results are compared for Liu and Jordan, Reindl, Hay and Badescu models finding maximum differences of about 1.5° between models, reported in December, in India.

Another numerical simulation of the optimum tilt for maximizing daily total solar energy received in Turkey, by applying Liu and Jordan model, is presented by Bakirci (2012). Considering a ground reflectance of 0.20 and correlating total terrestrial radiation to extraterrestrial one by the clearness index, the author presented numerical values for the optimum tilt and derived an expression function on solar declination δ proposing polynomial correlations valid for Turkey (e.g. $34.783-1.4317\delta-0.0081\delta^2+0.0002\delta^3$).

An analytical expression of the optimum tilt for maximum total solar energy computed with Liu and Jordan model is derived by Morcos (1994). The tilt is expressed in terms of azimuth angle, declination, latitude, ground reflectance, zenith and hour angles. The author derived also the analytical expression of the azimuth angle for maximum total energy and applied the results for Assiut (Egypt).

Hartley and Martinez-Lozano (1999) maximized solar irradiance on a horizontal plane, found a monthly average for this variation and computed the corresponding irradiance intercepted by a tilt collector using Liu and Jordan model, but without considering the view factor of the collector to the ground. The corresponding solar energy was obtained and numerical values for the optimum tilt were proposed.

From the above discussions, one might separate two optimization criteria: maximum intercepted solar energy (radiation, in J/m^2) and maximum intercepted solar irradiance (power, in W/m^2).

With respect to Hottel and Woertz model, Liu and Jordan forecast model takes into account the view factors to the sky and to the ground. In the current literature, the view factors calculation is of interest and proposed for application to solar radiation forecast (Sugden, 2004). In order to compute the radiation reflected by the ground or surface, ground albedo value is required. Psiloglou and Kambezidis (2009) reported several methods to calculate ground albedo underlining its important influence on the estimation of solar radiation incident on tilted surfaces. They have researched a methodology for deriving new site-specific ground-albedo expressions for locations without albedo measurements and compared the obtained reflected irradiance on a sloped surface to experimental measurements, emphasizing that a constant albedo of 0.20 is not convenient for Athens. Shamim et al. (2015) recently proposed a methodology for estimating clear sky global solar radiation, incorporating analytical tools for determining optical transmittance and surface albedo.

We have previously studied collectors optimum tilt calculation (Stanciu and Stanciu, 2014) and after applying a numerical simulation for the tilt at different latitudes, the following result was revealed: the correlation $\beta_{opt,noon}=\varphi-\delta$ applies for the optimum tilt when using Hottel and Woertz model for intercepting maximum solar irradiance at noon, but some deviations from this formula along the year appeared when using other forecast models (Liu and Jordan and Hay–Davis–Klucker–Reindl models). Up to 10° difference in the tilt value was obtained between models.

The purpose of this paper is to find analytical expressions for the optimum tilt, for further use in optimization procedures and