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Intensity of gravitational wave emitted by an oscillating Keplerian binary

M.N. Chowdhury[a,*], M.T.H. Bhuiyan[b], M.D.I. Bhuyan[c], S.B. Faruque[d]

[a] Department of Basic Sciences and Humanities, University of Asia Pacific, Dhaka, Bangladesh
[b] Department of Physics, Pabna University of Science and Technology, Pabna, Bangladesh
[c] Department of Physics, Mawlana Bhashani Science and Technology University, Santosh, Tangail-1902, Bangladesh
[d] Department of Physics, Shahjalal University of Science and Technology, Sylhet-3114, Bangladesh

HIGHLIGHTS

- Calculate intensity of gravitational wave for oscillating orbit.
- Calculate intensity of gravitational wave for oscillating circular orbit.
- Calculate intensity of gravitational wave for oscillating elliptic orbit.

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ABSTRACT

This paper attempts to formulate a way for calculating the intensity of gravitational wave from two point masses in Keplerian circular and elliptic orbits. The intensity is calculated with the assumption that the orbital plane of the binary undergoes small oscillation about the equilibrium x-y plane. This problem is simplification of a physically possible orbit where one of the point masses is spinning whereby the spin-orbit force drives the orbital plane to wobble in a complicated manner. It is shown that the total energy of gravitational wave emitted by the binary in this case is dominated by the parameters which take into account the oscillation of the plane. The results presented are in fact a generalization of the classic results of Landau and Lifshitz.

1. Introduction

Gravitational waves (GWs) have always attracted theoreticians and experimentalists in the field of cosmology and gravitation as a new tool to understand the universe. Scientists have been working vigorously ever since their prediction by the General Theory of Relativity to meet the challenge of detecting the ultra-weak quivers generated by these waves, and thereby unlocking the wealth of information contained in them about our universe and its evolution. As a result, the study of gravitational waves has become the focus of many physicists and lately there have been several works in both to develop theories as well as to improve the technology for detecting them. Earth-based laser-interferometric detectors of gravitational wave are now collecting data, and LIGO has just completed the longest scientific run (Abbott et al. 2016) to date and confirmed their existence. In such an exciting and important time of gravitational wave research, every bit of information on gravitational wave, be it theoretical, computational or experimental, is valuable. Binary star systems, consisting of compact objects such as black holes and neutron stars, are relatively strong sources of gravitational wave. Calculation of energy emitted due to gravitational wave by point masses in Keplerian elliptical orbit was performed by Peters and Mathews (1963); Landau and Lifshitz (1975). In current literature, the objectives of gravitational wave research are more focused on detection of GW (Grote 2008; Berti et al. 2008; Mendell and Wette 2008; Shoemaker et al. 2008) and on evolution of GW sources (Cutler, Kennefick and Poisson 1994; Ryan 1995; Glampedakis and Kennefick 2002; Gergely and Keresztes 2003). In present paper we consider the case that the orbital plane does not remain invariant on a plane. This is quite possible because in a binary where one of the bodies is spinning, the spin orbit-force drives the orbital plane to precession (Vecchio 2004) or to oscillate in a complex manner (Mashhoon and Singh 2006). Such precession or oscillation modulates the GW signal and the total energy emitted also changes. Here we consider a simplified problem from the scenario reported by Mashhoon and Singh (2006). Let two point masses in a Keplerian binary revolve round the center-of-mass in circular orbit and at the same time, the plane of the orbit is undergoing small oscillation about the

* Corresponding author. Fax: +8802-58157097.
E-mail address: nahian140@uap-bd.edu (M.N. Chowdhury).

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equilibrium x-y plane. We consider that the amplitude of angular oscillation about the x-y plane is very small compared to the radius of the orbit. We then calculate the energy emitted separately in the two polarization modes of gravitational wave and the total energy emitted i.e. intensity in all directions. We found that the amount of emitted energy depends on the nature of oscillation of the plane - in particular, the angular frequency of oscillation about the x-y plane. This is an important finding and we feel that many researchers would like to know the way to this result. The paper is organized as follows: In Section 2, we briefly summarize the important formulae of gravitational wave. Section 3 is about the review of the problem and represents the subsequent calculation of gravitational radiation from oscillating circular orbits and from oscillating elliptic orbits respectively of a Keplerian binary. Finally, Section 4 contains the conclusion.

2. Gravitational wave

Gravitational radiation emission from various astrophysical sources has been the focus of many researches (Zimmermann and Szedenits Jr 1999; Beltrami and Chau 1985; Dionysiou 1986; Shibata 1993; Moreno-Garrido, Buitrago and Mediavilla 1994; Moreno-Garrido, Buitrago and Mediavilla 1995; Blanchet 1996). Let us consider a source of gravitational radiation characterized by a mass quadrupole moment tensor $D_{\alpha\beta}$ with the six elements D_{xx}, D_{yy}, D_{zz}, D_{xy}, D_{yz}, D_{zx}, with respect to a set of fixed inertial axes (x,y,z). We define $D_{\alpha\beta}$ as by Landau and Lifshitz (1975), that is,

$$D_{\alpha\beta} = \int \rho\left(3x^{\alpha}x^{\beta} - \delta^{\alpha\beta}r^2\right)dV \tag{1}$$

where ρ is the mass density, and $r^2 = x^2 + y^2 + z^2$, $dV = dxdydz$. The waves can be taken to be plane in view of the typically large distance between the source and the observer. The two independent polarization states of the gravitational wave can be represented by the three-dimensional symmetric, unit polarization tensor $e_{\alpha\beta}$ satisfying the relations

$$e_{\alpha\alpha} = 0,\ e_{\alpha\beta}\ n_{\beta} = 0,\ e_{\alpha\beta}e_{\alpha\beta} = 1, \tag{2}$$

where $\hat{n}$ is a unit vector in the direction of propagation of the wave. Let us label the two polarizations by (Peters and Mathews 1963)

$$e_{+} = \frac{1}{\sqrt{2}}\left(\hat{\theta}\hat{\theta} - \hat{\varphi}\hat{\varphi}\right),\ e_{\times} = \frac{1}{\sqrt{2}}\left(\hat{\theta}\hat{\varphi} + \hat{\varphi}\hat{\theta}\right), \tag{3}$$

where θ and φ are conventional polar coordinates. In this basis, the waveform can be written as (Kochanek et al. 1990)

$$rh = (\ddot{D}_{\theta\theta} - \ddot{D}\varphi\varphi)e_{+} + 2\ddot{D}_{\theta}\varphi e_{\times}, \tag{4}$$

where h is the metric perturbation or the GW waveform, and $D_{\theta\theta}$, $D_{\theta\varphi}$, $D_{\varphi\varphi}$ are the physical components of D_{ij} (the Cartesian components of quadrupole tensor) projected along the directions of the spherical unit vectors $\hat{\theta}$ and $\hat{\varphi}$. There exists canonical procedure for obtaining these components, but we simply quote the results from Kochanek et al. (1990):

$$\begin{aligned} D_{\theta\theta} &= \left(D_{xx}cos^2\varphi + D_{yy}sin^2\varphi + D_{xy}sin2\varphi\right)cos^2\theta \\ &\quad + D_{zz}sin^2\theta - (D_{xz}cos\varphi + D_{yz}sin\varphi)sin2\theta, \\ D_{\varphi\varphi} &= D_{xx}sin^2\varphi + D_{yy}cos^2\varphi - D_{xy}sin2\varphi, \\ D_{\theta\varphi} &= -\frac{1}{2}(D_{xx} - D_{yy})cos\theta sin2\varphi + D_{xy}cos\theta cos2\varphi \\ &\quad + (D_{xz}sin\varphi - D_{yz}cos\varphi)sin\theta. \end{aligned} \tag{5}$$

The expressions for the intensity of radiation of a given polarization into solid angle $d\Omega$ are (Landau and Lifshitz 1975)

$$dI = \frac{G}{72\pi c^5}\left(\frac{d^3D_{\alpha\beta}}{dt^3}e_{\alpha\beta}\right)^2 d\Omega \tag{6}$$

where G is the Newton's gravitational constant and c is the speed of light in free space. Using Eqs. (3) and (4), we can write for the intensity of GW in $(\times)$ polarization as

$$\frac{dI_1}{d\Omega} = \frac{G}{72\pi c^5}\left(2\frac{d^3D_{\theta\varphi}}{dt^3}\frac{1}{\sqrt{2}}\right)^2 = \frac{G}{36\pi c^5}\left(\frac{d^3D_{\theta\varphi}}{dt^3}\right)^2, \tag{7}$$

and that in $(+)$ polarization as

$$\begin{aligned} \frac{dI_2}{d\Omega} &= \frac{G}{72\pi c^5}\left[\left(\frac{d^3D_{\theta\theta}}{dt^3} - \frac{d^3D_{\varphi\varphi}}{dt^3}\right)\frac{1}{\sqrt{2}}\right]^2 \\ &= \frac{G}{144\pi c^5}\left(\frac{d^3D_{\theta\theta}}{dt^3} - \frac{d^3D_{\varphi\varphi}}{dt^3}\right)^2 \end{aligned} \tag{8}$$

Next we apply these formulae to find out the intensity of gravitational wave emitted by a Keplerian binary whose orbital plane is oscillating about the equilibrium x-y plane.

3. Intensity of gravitational wave from a binary with oscillating orbital plane

In many astrophysical binary star systems, the orbit of the stars undergoes precession and oscillation due to many perturbing forces, such as, spin-orbit, spin-spin interactions. Specifically, the spin-orbit force drives the orbital plane to oscillate about the equilibrium plane in a quite complicated manner. One typical case is analyzed by Mashhoon and Singh (2006).

We consider a simplified situation defined by an almost fixed orbital plane confined to the x-y plane, but the orbital plane undergoes very small angular oscillation about the equilibrium x-y plane. This situation simulates some of the characteristics of orbital motion of a Keplerian binary with one particle having small spin. Now, we define the orbit by the following orbital variables:

$$r = constant,\ \theta = \frac{\pi}{2} - bsin\frac{\omega}{n}t,\ \varphi = \omega t \tag{9}$$

where ω is the Newtonian angular frequency of the orbit in the x-y plane, $r = |\vec{r_1} - \vec{r_2}|$; $\vec{r_1}$, $\vec{r_2}$ being the positions of the particles of mass m_1 and m_2, respectively, and b is a very small parameter $(b \ll 1)$ characterizing the angular oscillation about the x-y plane. Now to simplify, let us consider the frequency of oscillation of the orbital plane is same as the frequency of orbital motion (Mashhoon and Singh 2006). That is the time taken for one complete orbital motion is the same as the time taken for a complete oscillation of the plane. So ω is same for both. The complicated wobble motion can be represented by a single ω. But when the frequencies are not same, the resulted frequency of the complicated motion can be presented by $\frac{\omega}{n}$ where n is a parameter that relates the two frequencies. That is when $n = 1$, the two frequencies are same.

Now, we approximate the Cartesian components of the vector $\vec{r}$ as:

$$\begin{aligned} x &\cong rcos\omega t \\ y &\cong rsin\omega t \\ z &\cong rbsin\frac{\omega}{n}t \end{aligned} \tag{10}$$

Then, the quadrupole moments are:

$$\begin{aligned} &D_{xx} = \mu r^2\left(3cos^2\omega t - 1\right), \qquad D_{yy} = \mu r^2\left(3sin^2\omega t - 1\right), \\ &D_{xy} = \frac{3}{2}\mu r^2 sin2\omega t, \qquad D_{zz} = \mu r^2\left(3b^2sin^2\frac{\omega}{n}t - 1\right), \\ &D_{xz} = 3\mu br^2\left(cos\omega t sin\frac{\omega}{n}t\right),\ D_{yz} = 3\mu br^2\left(sin\omega t sin\frac{\omega}{n}t\right) \end{aligned} \tag{11}$$

where $\mu = \frac{m_1m_2}{m_1+m_2}$ is the reduced mass of the binary. Now, since the system is rapidly rotating about the z-axis, average over the angle φ is appropriate. Next, we take an average over the orbital