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Gravitational waves and magnetic monopoles during inflation with Weitzenböck torsion

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ABSTRACT

We study the variational principle on a Hilbert–Einstein action in an extended geometry with torsion taking into account non-trivial boundary conditions. We obtain an effective energy–momentum tensor that has its source in the torsion, which represents the matter geometrically induced. We explore about the existence of magnetic monopoles and gravitational waves in this torsional geometry. We conclude that the boundary terms can be identified as possible sources for the cosmological constant and torsion as the source of magnetic monopoles. We examine an example in which gravitational waves are produced during a de Sitter inflationary expansion of the universe.

1. Introduction

In the standard treatment of the variational principle over the Hilbert–Einstein action (HE), when a manifold has a boundary $\partial\mathcal{M}$, the action is supplemented by a boundary term which is in general neglected [1]. However, this is not the only manner to study this problem. As was recently demonstrated in [2], it is possible to include the flux around an hypersurface that encloses a physical source without the inclusion of extra terms in the HE action. In that paper was demonstrated that the non-zero flux of the vector metric fluctuations through the closed 3D Gaussian-like hypersurface, is responsible for the gauge-invariance of gravitational waves (GW). However, the torsional contributions were neglected in that paper. In the present paper we extend this analysis on the variational principle, but for an extended geometry with torsion. We obtain an effective energy–momentum tensor with sources only in torsion, which can be viewed as an effective matter tensor in a Riemannian geometry. Such tensor represents matter geometrically induced, but without extra dimensions. In addition, we develop a new manner to obtain GW on a torsional manifold taking into account nontrivial boundary terms. The first contribution to GW with purely torsional nature, was studied in Section 3 A, and the second one based in the boundary term was studied in Section 3 B; in both cases for a general torsion. Also, we present an example in Weitzenböck geometry obtaining the expression for the magnetic density monopoles and a gravitational wave equation for a Friedman-Robertson-Walker (FRW) in Weitzeböck geometry. Finally, in Section 6, we develop some final remarks.

2. Variational principle in torsional geometry

We consider the variational principle in presence of torsion for a HE-like action. We have studied this fundamental problem in [3]. Therefore, we shall expose some results in present section without making a full description. In [3], we have studied the contribution of the new terms which are not present in a Riemannian geometry. The boundary term was studied in [2], but emphasizing the role of non-metricity. Now, we shall consider some gravitational action in an extended geometry (i.e. a non-Riemannian manifold), without the presence of matter

$$I = \frac{1}{2\kappa}\int_M d^4x\sqrt{-g}R, \tag{1}$$

in which $\kappa = 8\pi G$, such that G is the gravitational constant, and

$$R^m_{\ lij} = \Gamma^m_{\ lj,i} - \Gamma^m_{\ li,j} + \Gamma^n_{\ lj}\Gamma^m_{\ ni} - \Gamma^n_{\ li}\Gamma^m_{\ nj}, \tag{2}$$

$$R_{lj} = R^i_{\ lij}, \tag{3}$$

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where $R = R_{nm}g^{nm}$ is the scalar curvature. We have employed the Einstein's convention over repeated indexes. The "," represents a partial derivative and all the indices run between 1 and 4. Furthermore, g_{ab} are the components of the metric tensor and $\sqrt{-g}$ is the volume of the non-Riemannian manifold. The Eq. (2) defines the Riemann curvature tensor, the Eq. (3) give us the Ricci tensor and Eq. (II) is the scalar curvature. We denote with Γ^a_{bc} an arbitrary affine connection, which is defined according to

$$\nabla_{\overrightarrow{e}_a} \overrightarrow{e}_b = \Gamma^n_{ba} \overrightarrow{e}_n, \tag{4}$$

where $\nabla_{\overrightarrow{e}_a}$ denotes the derivative in a-direction of the tangent space $\left\{\overrightarrow{e}_b\right\}$. Here, the up arrow means that the tangent space in the position representation is described by partial derivatives with respect to contravariant coordinates: $\left\{\overrightarrow{e}_b\right\} \equiv \left\{\frac{\partial}{\partial x^b}\right\}$, and the down arrow means that the cotangent space is generated by $\underset{\rightarrow}{e}^b \equiv \left\{dx^b\right\}$, such that $\underset{\rightarrow}{e}^b(\overrightarrow{e}_a) = \delta^b_a$. We wont consider any particular symmetry in the connections. Now we shall make the variation of the action in (1): $\delta I = 0$. Here we must take into account that the scalar R in (II) is related to the connection in (4), which is an abstract connection which is in general non-Riemannian, but fulfils the expression

$$\Gamma^n_{mr} = \left\{{}^{\,n}_{mr}\right\} + K^n{}_{mr}, \tag{5}$$

with $\left\{{}^{\,n}_{mr}\right\}$ the second kind Christoffel symbols representing the Riemannian or Levi-Civita connections, and K^n_{mr} the contortion tensor, which in absence of non-metricity is entirely torsional according to

$$K^a{}_{bc} = -\frac{g^{na}}{2}\{T^s{}_{cn}\, g_{bs} + T^s{}_{bn}\, g_{sc} - T^s{}_{cb}\, g_{sn}\}, \tag{6}$$

with the torsion tensor defined by

$$T^n_{mr} = \Gamma^n_{rm} - \Gamma^n_{mr}, \tag{7}$$

which is a valid expression in a coordinate basis of the four dimensional tangent space to the space–time manifold (TM4). In present work we impose the non-metricity free condition

$$N_{nmr} = g_{nm\,;r} = 0, \tag{8}$$

for an analysis of such contribution to the GW the reader can see [3]. The variation of the Ricci must be related to the variation of the connections obtaining a generalised Palatini identity for torsional geometry

$$g^{mr}\,\delta R_{mr} = W^n{}_{;n} - \frac{1}{2} g^{mr}(\delta\Gamma^n{}_{pr}\, T^p{}_{mn} + \delta\Gamma^n{}_{pm}\, T^p{}_{rn}), \tag{9}$$

with

$$W^n{}_{mr} = \delta\Gamma^n{}_{mr} - \delta\Gamma^k{}_{kr}\,\delta^n{}_m \tag{10}$$

where $W^n = g^{mr} W^n{}_{mr}$. With the use of Eq. (9) in the variation of the action we obtain

$$\delta I = \int_M d^4x\,\sqrt{-g}\left(R_{ab} - \frac{1}{2} R\, g_{ab}\right)\delta g^{ab} + \int_{\partial M} W^n\, d\Sigma_n - \frac{1}{2}\int_M d^4x\,\sqrt{-g}(\delta\Gamma^n_{pr}\, T^p_{mn} + \delta\Gamma^n_{pm}\, T^p_{rn}) g^{mr}. \tag{11}$$

In the first integral we recognize the Einstein tensor for the torsional connection. The second one is due to the boundary term. The third integral is completely originated by the torsion. This is a non-Riemannian contribution.

To finalize this section we must present the explicit form of the $W^n{}_{mr} = W^n{}_{(mr)} + W^n{}_{[mr]}$ tensor, where the symmetric and antisymmetric contributions are, respectively given by

$$\begin{aligned} W^n{}_{(mr)} = \Bigg[&\frac{g^{kn}}{2}\left\{\delta g_{mk\,,r} + \delta g_{kr\,,m} - \delta g_{rm\,,k} - T^t{}_{rk}\,\delta g_{mt} - T^t{}_{mk}\,\delta g_{tr}\right\} - \frac{\delta g^{kn}}{2}\left\{g_{mk\,,r} + g_{kr\,,m} - g_{rm\,,k} - T^t{}_{rk}\, g_{mt} - T^t{}_{mk}\, g_{tr}\right\} \\ &- \frac{g^{kl}}{4}\left(\delta g_{kl\,,r}\,\delta^n{}_m + \delta g_{kl\,,m}\,\delta^n{}_r\right) + \frac{\delta g^{kl}}{4}\left(g_{kl\,,r}\,\delta^n{}_m + g_{kl\,,m}\,\delta^n{}_r\right)\Bigg], \end{aligned} \tag{12}$$

$$W^n{}_{[mr]} = \left[\frac{g^{kn}}{2} T^t{}_{rm}\,\delta g_{tk} - \frac{\delta g^{kn}}{2} T^t{}_{rm}\, g_{tk} - \frac{g^{kl}}{4}\left(\delta g_{kl\,,r}\,\delta^n_m - \delta g_{kl\,,m}\,\delta^n{}_r\right) + \frac{\delta g^{kl}}{4}(g_{kl\,,r}\,\delta^n{}_m - g_{kl\,,m}\,\delta^n{}_r)\right], \tag{13}$$

such that $W^n = W^n{}_{(mr)}\, g^{mr}$.

3. Physics of the torsional geometry and 4D induced matter

In presence of torsion, but zero non-metricity, the variation of the action takes the form

$$\delta I = \int_M d^4x\,\sqrt{-g}\left[R_{ab} - \frac{1}{2} R\, g_{ab} - \frac{1}{2} L_{(ab)}\right]\delta g^{ab} + \int_M d^4x\,\sqrt{-g}\, W^n{}_{;n} \tag{14}$$

with

$$L_{(sd)} = \left\{\Delta^p{}_{mrsd}\, K^n{}_{pn} - \Delta^p{}_{nrsd}\, K^n{}_{pm} - \Delta^p{}_{nmsd}\, K^n{}_{pr} + \Delta^n{}_{npsd}\,(K^p{}_{rm} + K^p{}_{mr})\right\} g^{mr}. \tag{15}$$

Furthermore

$$\Delta^p{}_{mrsd} = \frac{g^{pk}}{2}\{-(g_{ms}\, g_{kd})_{,r} - (g_{ks}\, g_{rd})_{,m} + (g_{ms}\, g_{rd})_{,k} + T^l{}_{rk}\, g_{ms}\, g_{ld} + T^l{}_{mk}\, g_{ls}\, g_{rd} - T^l{}_{rm}\, g_{ls}\, g_{rd}\} - \frac{1}{2}(-T^l{}_{rd}\, g_{ml} - T^l{}_{md}\, g_{lr} + T^l{}_{rm}\, g_{ld})\,\delta^p_s. \tag{16}$$

The first integral in (14) includes the extended Einstein tensor with the torsional (Weitzenböck) contribution, and the second one includes the boundary contribution. We have obtained the expression (14) in absence of matter. We can distinguish two possible cases.

1. The first case describes infinity manifolds and there are no boundary contributions: $W^n_{;n} = 0$, so that the first integrand in (14) is null:

$$R_{ab} - \frac{1}{2} R\, g_{ab} = \frac{1}{2} L_{(ab)}. \tag{17}$$

After some algebraic manipulation of $L_{(ab)}$, the last assumption leads to a wave equation originated in the presence of torsion:

$$\Box\,(\Delta^n_{prsd}\,\delta g^{sd}\, T^p_{mn}) = 0, \tag{18}$$

where Δ^n_{prsd} is given by (16).

2. The second case describes finite manifolds so that the boundary contributions are significative: $W^n_{;n} \neq 0$. In that case the first integrand must be nonzero in order to $\delta I = 0$:

$$\delta g^{ab}\left[R_{ab} - \frac{1}{2} R\, g_{ab} - \frac{1}{2} L_{(ab)}\right] + W^n{}_{;n} = 0. \tag{19}$$

In the relativistic formalism without boundary conditions (i.e., when $W^n{}_{;n} = 0$), the cosmological constant can be added to the Einstein equations as an integration constant. Therefore,