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The semi-Markov model for the 'technological module–storage device' structure

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Abstract

The theory of semi-Markov processes has been used to design a model of a 'technological module–storage device' (TM–SD) structure. Stationary characteristics based on the obtained equations were determined to find a stationary distribution of the Markov embedded chain. Relying upon the performed studies, the stationary distribution of a semi-Markov process was determined. This allowed calculating the availability ratio of the TM–SD structure, and the design formula was given. The Markov restoration equations for the TM–SD system with taking into account TM and SD failures were solved assuming the exponential behavior of these failures. The obtained expressions describe how such a system operates and allow substituting the TM–SD system with an equivalent element with two factor states. This result significantly simplifies the modeling problem for more complex systems. The legitimacy of using exponential distributions of random variables (error-free periods for TM and SD) was analyzed. The performed simulation modeling revealed that the hypothesis for an exponential behavior of error-free periods for TM as a whole (and SD as well) can be accepted even in the case when TM (or SD) consists of six nodes.

Keywords: Semi-Markov model; Markov restoration equation; Embedded Markov chain; Stationary distribution.

Introduction

Time redundancy still remains one of the most effective most effective methods for improving the reliability of technical devices, especially of asynchronous automated lines whose technological modules are connected to each other through storage devices. A considerable number of studies [1–7] is dedicated to this problem; the authors of many of these works limit the discussion to finding the availability of the system under consideration. Let us note, however, that a hierarchical approach to constructing models of complex stochastic systems [8–11] is frequently used; such an approach involves having to fit individual elements of the

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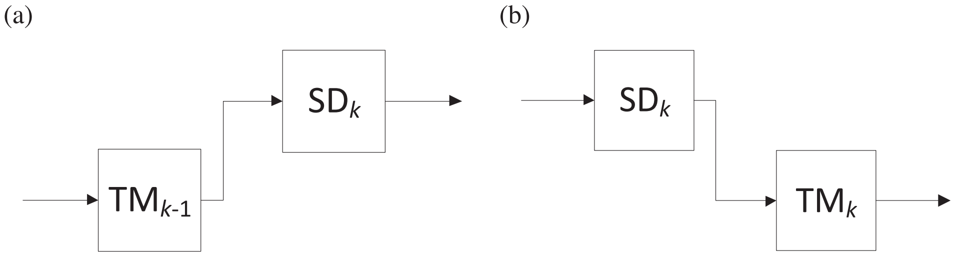

Fig. 1. 'Technological module (TM)–storage device (SD)' structure working to yield (a) and receive (b) the production; k is the index number.

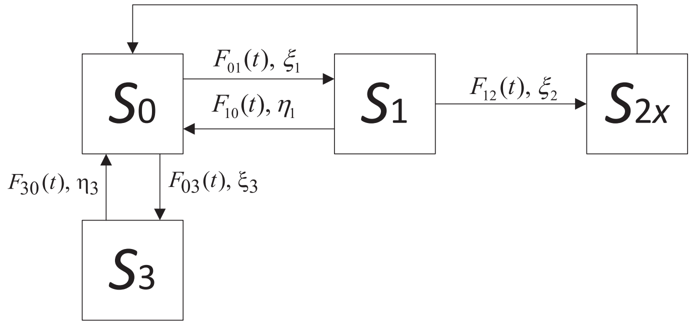

Fig. 2. Graph of the states of the TM–SD system (see Table 1).

system to one another. For this purpose, it is necessary to know the distribution functions of the mean time to failure and the mean time to repair for these elements. Another drawback of the proposed models is ignoring the reliability of the storage device [12–14], since it significantly complicates the problem. However, when developing models of multiphase systems, functions of the distributions of the mean times between failure and recovery of elements (MTBF, and MTTR, respectively) with time redundancy should be used. In this case, the time reserve should generally depend on the storage device's reliability.

The goal of this study is to develop a semi-Markov model describing how the 'technological module–storage device' structure operates and allowing to determine its availability and the above-noted distribution functions for the structure as a whole. In practice, this means that this structure can be equivalently replaced by a primitive element with two factor states, downtime and uptime.

Fig. 1 shows possible structures of the technological modules (TMs) with intermediate storage devices (SDs) providing time redundancy (the structures operate as a part of an asynchronous automated line).

The difference between the structures in Fig. 1 is due to the fact that the same SD can work together with the preceding TM to yield production and with the subsequent TM to receive it. From a mathematical standpoint, the structures presented in Fig. 1(a) and (b) are identical, so the model shall be constructed only for the case in Fig. 1(a).

Problem setting

The problem can be formulated in the following way: let us assume that the distribution functions $F_{01}(t)$ and $F_{10}(t)$ of the random variables ξ_1 and η_1 (which are the mean times between TM failure and recovery, respectively) are known, along with the distribution functions $F_{03}(t)$ and $F_{30}(t)$ of the random variables ξ_3 and η_3 (which are the mean times between SD failure and recovery, respectively). Additionally, the distribution function $F_{12}(t)$ of the random variable ξ_2 which is the reserve time is also known.

Let us make the following assumptions:

The probability of simultaneous failures of the TM and the SD is neglected (since this event is improbable);

The distribution functions $F_{01}(t)$ и $F_{03}(t)$ take the exponential form.

For the purposes of our study, it is necessary to determine the distribution functions of for the mean times between failure and recover of the area as a whole, i.e., to equivalently replace it by the primitive element with two factor states.