Current Applied Physics 16 (2016) 1459–1463

Contents lists available at ScienceDirect

Current Applied Physics

journal homepage: www.elsevier.com/locate/cap

Fluid-induced resonances in vibrational and Brownian dynamics of a shear oscillator

Manhee Lee [1], JongGeun Hwang, Bongsu Kim, Sangmin An, Wonho Jhe *

Department of Physics and Astronomy, Institute of Applied Physics, Seoul National University, Seoul, 151-747, South Korea

A R T I C L E   I N F O

Article history:
Received 13 July 2016
Accepted 13 August 2016
Available online 16 August 2016

Keywords:
Atomic force microscopy
Oscillator
Brownian motion
Hydrodynamics

A B S T R A C T

We generally describe vibrational and Brownian dynamics of a shear oscillator interacting with a confined Newtonian fluid. We show that the shear oscillator exhibits three characteristic dynamics in viscous, weak inertial, and strong inertial regimes, and the dynamics are controlled by two system parameters, effective confining height and oscillator-fluid coupling strength. While resonances of oscillators are usually deteriorated in fluids, we interestingly find the resonances arisen in originally overdamped oscillators, originated from hydrodynamic memory effects in strong inertial regime. The present theory could be exploited for improving designs and performances of shear oscillators in fluids, and reciprocally for investigating fluid properties by using shear oscillatory probes such as atomic force microscope in shear-mode.

1. Introduction

The mechanical oscillator is a fundamental tool in science and technology for measuring physical interactions ranging from atomistic quantum forces [1] to cosmological gravitational waves [2]. In particular, oscillators are typically employed as sensors in micro-electromechanical system (MEMS) [3] and scanning probe microscopy (SPM) [4]. With the increased utilization of MEMS and SPMs in nano-bio technology, liquid-immersed MEMS and SPMs are emerging in a range of applications. Examples include environmental sensing [5,6], bio-chemical mass detection [7], single molecule force spectroscopy [8], and visualizing biomolecular processes [9] in liquids. These applications require sensitive and precise measurement of the external forces exerted on the oscillators, which can be extracted from their dynamic response in fluids. Therefore, as evident by recent attention [10], it is important to understand the dynamic responses of mechanical oscillator interacting with fluids quantitatively.

An oscillator's dynamics in a fluid is highly coupled to the fluid hydrodynamics. The kinetic motion of the oscillator is damped by the viscous drag of the fluid, producing a flow around the oscillator. The entrained flow then acts back on the oscillator, exerting in turn a force on it. This oscillator-fluid 'coupled' nature strongly influences the dynamics and performance of the oscillator, depending on the time scale of the motions and the detailed geometry of the system. For the tapping-mode oscillator, oscillating normal to a nearby solid wall, the 'coupled' mechanics has long been comprehensively studied in vibrational [11–13] and in Brownian oscillators [14]. On the other hand, for shear oscillators vibrating parallel to the wall, the traditional Stoke's problem well describes the shear hydrodynamics of the confined fluid itself [15], while classical lubrication theory solves the shear force between two sliding surfaces neglecting the fluid inertia [16]. Although theses theories provide critical understanding of the sheared fluid dynamics and associated interaction, detailed dynamics of the shear oscillator coupled to the fluid has not yet completely understood.

In this paper, we present the general dynamics solution for a shear oscillator coupled to Newtonian fluid near a surface, taking into account fully the fluid's inertia and viscosity. We provide a holistic understanding of the coupled shear interaction and the resultant three types of oscillator's dynamics, as summarized in the phase map. Interestingly, we find resonances in vibrational and Brownian motions of originally overdamped oscillators, induced by hydrodynamic memory effect. These results not only could be of practical use for efficient operation of shear SPM and MEMS in measuring the shear friction and stress [17] and in sensitive imaging in fluids [9], but also offer a novel way to investigate material properties using shear oscillators.

* Corresponding author.
E-mail address: whjhe@snu.ac.kr (W. Jhe).
[1] Present Address: S. LSI, Giheung-campus, Samsung Electronics, Yonginsi, 446–811, South Korea.

http://dx.doi.org/10.1016/j.cap.2016.08.007
1567-1739/

2. Shear interactions mediated by a confined fluid

The coupled system of a shear oscillator and a confined fluid can be modeled as shown in Fig. 1(a), where the upper plate represents the oscillator and laterally oscillates on a fluid. This driven oscillator interacting with the fluid is described as

$$m\ddot{x} + b\dot{x} + kx = Fe^{iwt} + F_s, \quad (1)$$

where m is the effective mass of the probe, b the intrinsic damping coefficient, k the spring constant of the probe, Fe^{iwt} the driving force, w the driving angular frequency, and F_s the fluid-induced shear force. Under no-slip boundary condition at the probe-fluid interface, the shear force is given by the product of the contact area σ and the stress tensor τ_{ij} at the interface, i.e., $F_s = -\sigma\tau_{xz}(h) = -\sigma\eta\, \partial U/\partial z|_{z=h}$, where η is the viscosity and U the velocity field of the fluid in the x-direction. Here only the xz-component of τ_{ij} is considered due to the symmetry. The fluid motion $U(z,t)$ is determined by Navier-Stokes equation for an incompressible Newtonian fluid with the symmetry [18],

$$\frac{\partial U}{\partial t} = \frac{\eta}{\rho}\frac{\partial^2 U}{\partial z^2}, \quad (2)$$

where ρ is the fluid density. The oscillator-fluid dynamics (Fig. 1(a)) is generally governed by the coupled equations of Eqs. (1) and (2) with no-slip boundary conditions at the upper and lower interfaces, $U(z=h,t) = \dot{x}(h,t)$ and $U(z=0,t)=0$.

In the present oscillatory system, we look for periodic solutions for both the probe motion $x(h,t)$ and the fluid flow $U(z,t)$ such as,

$$U(z,t) = u\, e^{i(wt+\theta)}, \quad (3)$$

$$x(h,t) = Ae^{i(wt+\theta-\pi/2)}. \quad (4)$$

Substituting Eq. (3) into Eq. (2), we obtain the velocity profile of the fluid,

$$\overline{u} \equiv \frac{U}{wx} = i\frac{\sin\left\{(1-i)\overline{h}\sqrt{\overline{w}}\ z/h\right\}}{\sin\left\{(1-i)\overline{h}\sqrt{\overline{w}}\right\}}, \quad (5)$$

where $\overline{h} = h/\delta_0$, $\delta_0 = \sqrt{2\eta/(w_0\rho)}$, $\overline{w} = w/w_0$, and $w_0=k/m$. Then, the spatial gradient of the fluid velocity field (Eq. (5)) gives the fluid-mediated shear force,

$$\overline{F_s} \equiv F_s\Big/\left(\kappa_0\overline{w}^{3/2}x\right) = -(1+i)\cot\left\{(1-i)\overline{h}\sqrt{\overline{w}}\right\}, \quad (6)$$

where $\kappa_0 = \sigma w_0^{3/2}\sqrt{\eta\rho/2}$.

Here two characteristic parameters δ_0 and κ_0 were used to describe the fluid velocity and the interaction force. The characteristic length δ_0, called 'the depth of penetration' [18], is the thickness of the fluid layer where the viscosity effect is dominant from the confining surface. As shown in Fig. 1(c), the interaction force F_s exhibits the usual viscous drag force proportional to $1/h$ at $h<\delta_0$ with $w=w_0$, while it remains constant at $h>\delta_0$ (Eq.(6)). The constant value, F_s at $h>\delta_0$, is given by $F_s \approx 2\kappa_0 A$. Thus, κ_0 serves as the characteristic force constant of the fluid-mediated shear interaction.

The interaction force F_s can be represented by the sum of anti-elastic and viscous damping forces. Notice that the fluid velocity $\overline{u}$ has the real and imaginary components, $\overline{u} \equiv \overline{u_x} + i\overline{u_v}$. From the definition of $\overline{u}$ ($\equiv U/(wx)$), $\overline{u_x}$ corresponds to a flow proportional to the displacement of the plate, while $\overline{u_v}$ to the velocity ($v=iwx$). The resulting interaction force, which is given by the spatial gradient of $\overline{u_x}$ and $\overline{u_v}$, is therefore written as the sum of position-dependent elastic force and velocity-dependent viscous force, $F_s=-k_s x - ib_s wx$, where we define effective elasticity k_s and damping coefficient b_s of the interaction. Notice that the elastic and

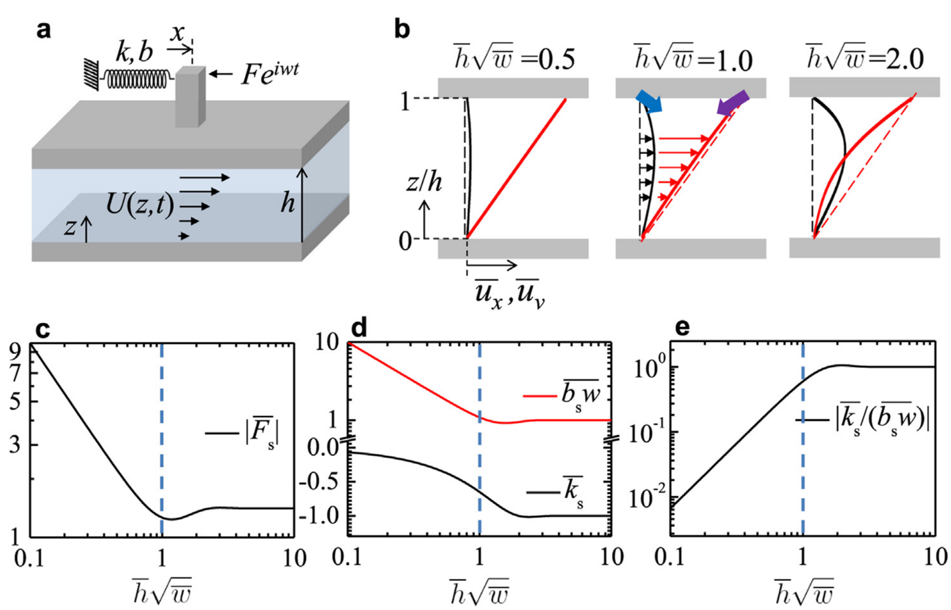

Fig. 1. Shear interaction mediated by a confined fluid. (a) Schematic diagram of the coupled oscillator-fluid system, where the mechanical oscillator is under shear motion above the fluid. (b) Numerical spatio-velocity profiles of the shearing fluid versus the normalized confining height $\overline{h}\sqrt{\overline{w}}$. As the confining distance or the shear frequency, or equivalently $\overline{h}\sqrt{\overline{w}}$, increases, there arises a nonlinear inertial flow ($\overline{u_x}$), black curves) besides the usual linear viscous flow ($\overline{u_v}$, red curves). The fluid-mediated forces exerted on the upper plate, proportional to the velocity gradient, are represented by the blue and purple arrows. (c) The magnitude of the normalized total interaction force $\overline{F_s}$ (Eq. (6)). (d) Decomposition of $\overline{F_s}$ into the inertial ($\overline{k_s}$) and viscous damping ($\overline{b_s w}$) interactions, corresponding to the real and imaginary parts of $\overline{F_s}$, respectively. (e) Ratio of inertial to viscous forces, $|\overline{k_s}/\overline{b_s w}|$, versus $\overline{h}\sqrt{\overline{w}}$. While $\overline{b_s w}$ dominates in viscous regime ($\overline{h}\sqrt{\overline{w}} < 1$), $\overline{k_s}$ and $\overline{b_s w}$ exhibit the same magnitude in inertial regime ($\overline{h}\sqrt{\overline{w}} > 1$) [19]. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)