Journal of Magnetism and Magnetic Materials 421 (2017) 152–164

Contents lists available at ScienceDirect

Journal of Magnetism and Magnetic Materials

journal homepage: www.elsevier.com/locate/jmmm

Magnetohydrodynamic stability of natural convection in a vertical porous slab

B.M. Shankar [a,*], Jai Kumar [b], I.S. Shivakumara [c]

[a] Department of Mathematics, PES University, Bangalore, 560 085 India
[b] ISRO Satellite Centre, Bangalore, 560 017 India
[c] Department of Mathematics, Bangalore University, Bangalore, 560 056 India

ARTICLE INFO

Article history:
Received 2 March 2016
Received in revised form
25 June 2016
Accepted 2 August 2016
Available online 3 August 2016

Keywords:
Stability
Magnetic field
Natural convection
Vertical porous layer

ABSTRACT

The stability of the conduction regime of natural convection in an electrically conducting fluid saturated porous vertical slab is investigated in the presence of a uniform external transverse magnetic field. The flow in the porous medium is described by modified Brinkman-extended Darcy equation with fluid viscosity different from effective viscosity. The boundaries of the vertical porous slab are assumed to be rigid-isothermal and electrically non-conducting. The resulting stability equations are solved numerically using Galerkin method. The critical Grashof number G_c, the critical wave number α_c and the critical wave speed c_c are computed for a wide range of porous parameter σ_p, the ratio of effective viscosity to the fluid viscosity Λ, the Prandtl number Pr and the Hartmann number M. Based on these parameters, the stability characteristics of the system are discussed in detail. The presence of advective inertia is to instill instability on the flow in a porous medium and found that the magnetic field, porous parameter and ratio of viscosities have a stabilizing effect on both stationary and oscillatory wave instabilities. Besides, the value of Pr at which transition occurs from stationary to oscillatory mode of instability decreases with increasing M, σ_p and Λ.

1. Introduction

The stability of natural convection in a vertical fluid layer whose side walls are maintained at different fixed temperatures is regarded as one of the classical problems in fluid mechanics and has been studied experimentally, analytically and numerically [1–5]. This work has subsequently been extended to include various additional effects and one such extension work undertaken was to understand the effect of transverse magnetic field on the stability of natural convection in a vertical electrically conducting fluid layer. The stability of fully developed pressure-driven steady flow of an electrically conducting fluid between two infinite parallel plates was first studied theoretically by Lock [6] for the case when a uniform external magnetic field perpendicular to the plates is present. Whereas, Takashima [7] was the first to examine the linear stability of natural convection of an electrically conducting fluid which is confined between two parallel vertical plates maintained at different constant temperatures in the presence of a transverse magnetic field. Potter and Kutchey [8] investigated stability of plane Hartmann flow subject to a transverse magnetic field while Takashima [9] re-examined the same problem as the boundary conditions on the magnetic field perturbations were incorrect. He solved the problem numerically using Chebyshev collocation method with the corrected boundary conditions. The linear stability of plane-Poiseuille flow at high Reynolds numbers and in the presence of a transverse magnetic field is investigated theoretically using the multideck asymptotic approach by Makinde [10], while Makinde and Mhone [11] studied the temporal stability of magnetohydrodynamic Jeffery-Hamel flows at very small magnetic Reynolds number. Proskurin and Sagalakov [12] studied the stability of Poiseuille flow in the presence of a longitudinal magnetic field and investigated about the dependence of the critical Reynolds number on the electrical conductivity. At large Reynolds numbers they found a new branch of instability and a sudden change in the critical Reynolds number. Makinde and Mhone [13] investigated numerically the temporal development of small disturbances in a channel filled with a saturated porous medium under the influence of magnetic field, while Makinde and Onyejekwe [14] considered the steady flow and heat transfer between parallel insulated walls with an applied transverse magnetic field. Belyaev and Smorodin [15] studied the linear stability of the convective flow in a vertical layer of ferrofluid subject to a transverse temperature gradient and a uniform

* Corresponding author at: Department of Mathematics, PES University, Bangalore 560 085, India.
E-mail address: bmshankar@pes.edu (B.M. Shankar).

http://dx.doi.org/10.1016/j.jmmm.2016.08.010
0304-8853/

Nomenclature

A	ratio of heat capacities
B_x, B_y, B_z	components of magnetic induction $\vec{B}$
c	wave speed
c_r	phase velocity
c_i	growth rate
$\vec{g}$	acceleration due to gravity
G	Grashof number
h	thickness of the porous slab
$\hat{i}$	unit vector in x-direction
k	permeability
$\hat{k}$	unit vector in z-direction
M	Hartmann number
P	total pressure
Pr	Prandtl number
Pr_m	magnetic Prandtl number
u, v, w	components of velocity vector $\vec{q}$
t	time
T	temperature
T_1	temperature of the left vertical rigid boundary
T_2	temperature of the right vertical rigid boundary
(x, y, z)	Cartesian co-ordinates

Greek symbols

α	vertical wave number
β	horizontal wave number
α_T	volumetric thermal expansion coefficient
β_T	temperature gradient
ε	porosity of the porous medium
θ	amplitude of perturbed temperature
κ	thermal diffusivity
Λ	ratio of effective viscosity to the fluid viscosity
μ	magnetic permeability
μ_e	effective viscosity
μ_f	fluid viscosity
ν	kinematic viscosity
ρ	fluid density
ρ_0	reference density at T_0
σ	electrical conductivity
σ_p	porous parameter
ϕ	velocity stream function
ψ	magnetic stream function

magnetic field directed normally to the layer boundaries using the Langevin law of magnetization. Adesanya et al. [16] studied the free convective flow of magnetohydrodynamic fluid through a channel with time periodic boundary conditions by taking the effects of Joule dissipation into consideration. Other important work on hydromagnetic fluid flow includes [17–23].

Copious literature is also available related to the work on the stability of natural convection in a viscous fluid saturated vertical porous layer whose walls are kept at different constant temperatures. Gill [24] was the first to investigate analytically the stability properties of convection in a vertical layer of Darcy porous medium for which one boundary is uniformly cold and the other is uniformly hot. A fully nonlinear analysis was considered on the Gill's stability problem by Straughan [25] and Flavin and Rionero [26]. In particular, Straughan [25] showed that Gill's proof of stability can be extended to the nonlinear domain of perturbations. Kwok and Chen [27] investigated separately the effect of no-slip boundary conditions for velocity, implemented by Brinkman's model of momentum balance instead of Darcy's law, and the effect of temperature-dependent viscosity within Darcy's law considering the basic flow states differ from that considered by Gill [24]. These authors found that, within a linear stability analysis, both no-slip conditions and variable viscosity are able to yield instability and, hence, to modify the conclusion implied by Gill's proof. Later Lewis et al. [28] quantified Gill's results by determining how quickly the disturbances decay. Rees [29] considered the effects of local thermal nonequilibrium on the stability of convection in a vertical porous channel by adopting the method used by Gill [24] while Scott and Straughan [30] performed the nonlinear stability analysis of the problem.

The effect of horizontal AC electric field on the stability of natural convection in a vertical dielectric non-Newtonian/Newtonian fluid layer (Takashima and Hamabata [31], Shankar et al. [32]) and also in a dielectric fluid saturated vertical porous layer has been investigated (Shankar et al. [33]). Nonetheless, it is intriguing to note that the buoyancy-driven convection in an electrically conducting fluid saturated porous medium under the influence of magnetic field is important from the viewpoint of its natural occurrence and in many engineering applications. For example, the study of the interaction of geomagnetic field with an electrically conducting fluid is of great interest in geophysics to study the Earth's core and to understand the performance of petroleum reservoir (Wallace et al. [34]). Besides, there have been experimental studies available on the subject of magnetoconvection in a mushy layer (Bergman and Fearn [35], Bergman et al. [36]). Bergman and Fearn [35] explained convection in a mushy zone and in the presence of a magnetic field and concluded that strong magnetic field can adversely affect the chimney formation in the mush. Bergman et al. [36] provided experimental evidence that the presence of a magnetic field can significantly reduce the chimney formation in a mushy layer. Riahi [37] briefly discussed experimental evidences concerning magnetoconvection in porous media. Work has also been carried out theoretically but studies are mainly concentrated on Benard type of instability in a horizontal porous layer in the presence of a uniform vertical magnetic field [38–45].

The stability of natural convection in an electrically conducting fluid saturated vertical Brinkman porous layer under the imposed magnetic field has not been previously investigated despite its importance in many practical problems as noted above. The study is conducted through a linear stability analysis and the resulting generalized eigenvalue problem is solved numerically. The impact of transverse magnetic field and the influence of other physical parameters on the modes of instability have been discussed in detail. The plan of the paper is as follows. Section 2 presents the mathematical formulation of the problem; Section 3 contains details of the linear stability analysis. Squire's transformation has been established in Section 3.1. Section 4 discusses the numerical solution for the present study. Results and discussion are presented in Section 5. Finally, the conclusion gives a summary of what has been investigated in this study.

2. Mathematical formulation

The physical configuration is as shown in Fig. 1. We consider an incompressible electrically conducting fluid saturated vertical layer of Brinkman porous medium in the presence of a uniform