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A constitutive model for magnetostriction based on thermodynamic framework

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ABSTRACT

This work presents a general framework for the continuum-based formulation of dissipative materials with magneto–mechanical coupling in the viewpoint of irreversible thermodynamics. The thermodynamically consistent model developed for the magnetic hysteresis is extended to include the magnetostrictive effect. The dissipative and hysteretic response of magnetostrictive materials is captured through the introduction of internal state variables. The evolution rate of magnetostrictive strain as well as magnetization is derived from thermodynamic and dissipative potentials in accordance with the general principles of thermodynamics. It is then demonstrated that the constitutive model is competent to describe the magneto-mechanical behavior by comparing simulation results with the experimental data reported in the literature.

1. Introduction

Magnetostrictives can be classified as smart materials exhibiting the conversion of energy type between input and output. The study on the magnetostrictive effect has been performed since James Joule first observed the length change in a sample of iron during the process of magnetization. Magnetostrictive materials convert magnetic energy into mechanical energy and vice-versa, which provides actuation and sensing capabilities in a wide range of device applications. Moreover, the knowledge of magneto-mechanical effect in the design of electrical machines plays important role in regard to the reduction of noise and vibration. The recent increase of interest in magnetostrictives caused by the commercial availability of giant magnetostrictive materials makes it necessary to develop a constitutive model that can more accurately predict nonlinear, hysteretic and dissipative magnetostriction in order to optimize the design of magneto-mechanical devices.

Magnetostrictive materials exhibit mechanical deformation in response to magnetic fields and change their magnetization state when mechanically stressed. Due to the coupling effect between magnetization and magnetostriction, the applied mechanical stress thus influences on both of magnetic and magnetostrictive hysteresis. It is well known that there exist two kinds of magnetostriction: spontaneous and field-induced magnetostrictions [1,2]. When a ferromagnetic material is cooled through its Curie temperature, it undergoes dimensional changes due to the magnetic phase transition from disordered magnetic states to ordered ones, which is known as spontaneous magnetostriction. The resulting volume of the ordered magnetic moments is called a magnetic domain. All magnetic moments in a domain are aligned parallel and the direction of the moments randomly varies from domain to domain throughout the ferromagnetic material. The bulk magnetization is thus considered to be zero. On the other hand, the field-induced magnetostriction is mainly attributed to the reorientation of magnetic moments associated with external magnetic fields. In general applications, magnetostrictive materials are utilized in the demagnetized and ordered state. The present work accordingly intends to propose a constitutive model for the field-induced magnetostriction, not for spontaneous one. It is also known that magnetostrictive materials can expand or contract in the direction of magnetization. Compressive mechanical stress decreases the magnetization of materials expanding under external magnetic field, which are called as positive magnetostriction, whereas tensile stress does the opposite. In contrast, the materials of negative magnetostriction contracting in the direction of magnetic field exhibit the magnetization property increased with compressive mechanical stress [2–4].

For several decades, a wide variety of constitutive models have been developed to describe the nonlinear magnetostrictive effect. In essence, numerous modeling works are based on the framework of the Preisach and Jiles–Atherton models. Some approaches on magneto-mechanical effect seem to follow the phenomenological description as in the Preisach model [5–7]. Others, meanwhile, utilize the concept of the Jiles–Atherton model in the context of physically based energy aspects [8–11]. Another class of constitutive models makes use of the thermodynamic principles [12]. These thermodynamically motivated models intend to describe the magnetostrictive material response through the definition of a specific free energy function in the Taylor series form [13–15] or through the introduction of internal state variables [16–18].

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In the present work a thermodynamically consistent model is proposed to account for the hysteretic properties of magnetization and magnetostriction in magnetostrictive materials as an extension of the previous model [19] developed for the magnetic hysteresis. The proposed model is not intended to explain the microscopic mechanisms for the magneto-mechanical behavior. The focus of this work is the phenomenological modeling of the macroscopic material response in the framework of thermodynamics. The formulation of the non-linear and dissipative nature is based on the thermodynamic consistency satisfying the first and second laws of thermodynamics. The internal state variables are introduced to characterize the non-equilibrium state from the viewpoint of the irreversible thermodynamics. The magnetostrictive strain is described through its evolution rate deriving from thermodynamic and dissipative potentials.

2. Constitutive theory

The first and second principles of thermodynamics are used to derive the constitutive relations of magnetostrictive materials undergoing magneto-mechanical loading; the first law states the conservation of energy and the second law restricts the direction of thermodynamic process. Moreover, the thermodynamic formalism is based on the postulations of two potentials characterizing all thermodynamic properties of a material. One referred to as thermodynamic potential is needed to mainly describe the reversible process, while the other is a dissipation potential needed for the irreversible behavior. Following the methodology for representation of dissipative effects by Coleman and Gurtin [20], the constitutive equations are finally completed in the form of evolution rates for the state variables.

2.1. Thermodynamic framework

On the assumption of small deformation, the local form of the first and second laws of thermodynamics can be written as follows:

$$\rho\dot{u} = \mathbf{H}\cdot\dot{\mathbf{B}} + \boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}} + r - \nabla\cdot\mathbf{q} \tag{1}$$

$$\rho\dot{s} + \mathrm{div}\frac{\mathbf{q}}{T} - \frac{r}{T} \geq 0 \tag{2}$$

where ρ is mass density, u the specific internal energy, r the internal heat source per unit mass, $\mathbf{q}$ the heat flux vector, s the specific entropy, and T the temperature. $\mathbf{H}$ and $\mathbf{B}$ stand for the magnetic field strength and magnetic flux density vectors, respectively, and $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ for the second-order stress and strain tensors. A superposed dot designates differentiation with respect to time. The dot and colon represent the scalar products; $\mathbf{H}\cdot\mathbf{B} = H_iB_i$ and $\boldsymbol{\sigma}:\boldsymbol{\varepsilon} = \sigma_{ij}\varepsilon_{ij}$. The combination of the first and second laws through the elimination of the internal heat source results in the following Clausius–Duhem inequality:

$$\rho(T\dot{s} - \dot{u}) + \mathbf{H}\cdot\dot{\mathbf{B}} + \boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}} - \mathbf{q}\cdot\frac{\nabla T}{T} \geq 0 \tag{3}$$

The present work makes use of the specific Helmholtz free energy as the thermodynamic potential. The free energy is defined by

$$\pi = u - Ts \tag{4}$$

Using Eqs. (3) and (4), the Clausius–Duhem inequality is rewritten as follows:

$$\mathbf{H}\cdot\dot{\mathbf{B}} + \boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}} - \rho(\dot{\pi} + s\dot{T}) - \mathbf{q}\cdot\frac{\nabla T}{T} \geq 0 \tag{5}$$

We assume the additive decomposition of the total magnetic flux density:

$$\mathbf{B} = \mathbf{B}^{rev} + \mathbf{B}^{irr} \tag{6}$$

where $\mathbf{B}^{rev}$ and $\mathbf{B}^{irr}$ are the reversible and irreversible parts, respectively. In addition, the total strain is decomposed into two parts:

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^{rev} + \boldsymbol{\varepsilon}^{mag} \tag{7}$$

where $\boldsymbol{\varepsilon}^{rev}$ is the reversible elastic strain and $\boldsymbol{\varepsilon}^{mag}$ denotes the magnetostrictive strain. Conventionally, the magnetostrictive deformation is assumed to be incompressible, viz. $tr(\boldsymbol{\varepsilon}^{mag}) = \varepsilon_{ii}^{mag} = 0$. The constitutive theory follows the approaching scheme of Coleman and Gurtin who introduced the method of utilizing internal state variables to account for dissipative effects in a physical system, which results in the hysteresis phenomenon of the system response. The thermodynamic potential is thus assumed to depend on a set of internal state variables as well as the observable state variables. Taking such a viewpoint, the dependence of the Helmholtz free energy is assumed to be

$$\pi = \pi\left(\mathbf{B},\ \mathbf{B}^{irr},\ \boldsymbol{\varepsilon}^{rev},\ T,\ \boldsymbol{\xi}_1\right) \tag{8}$$

where $\boldsymbol{\xi}_1$ is an internal state variable vector.

By substituting the time derivative of the free energy with respect to its independent state variables into Eq. (5), the Clausius–Duhem inequality is written as

$$\left(\mathbf{H} - \rho\frac{\partial\pi}{\partial\mathbf{B}}\right)\cdot\dot{\mathbf{B}} + \left(\boldsymbol{\sigma} - \rho\frac{\partial\pi}{\partial\boldsymbol{\varepsilon}^{rev}}\right):\dot{\boldsymbol{\varepsilon}}^{rev} - \rho\frac{\partial\pi}{\partial\mathbf{B}^{irr}}\cdot\dot{\mathbf{B}}^{irr} + \boldsymbol{\sigma}:\dot{\boldsymbol{\varepsilon}}^{mag} - \rho\left(s + \frac{\partial\pi}{\partial T}\right)\dot{T} - \rho\frac{\partial\pi}{\partial\boldsymbol{\xi}_1}\cdot\dot{\boldsymbol{\xi}}_1 - \mathbf{q}\cdot\frac{\nabla T}{T} \geq 0 \tag{9}$$

Each term in parentheses must vanish independently since the inequality holds for arbitrary changes of the independent state variables ($\mathbf{B}$, $\boldsymbol{\varepsilon}^{rev}$, T), leading to the relations

$$\mathbf{H} = \rho\frac{\partial\pi}{\partial\mathbf{B}},\quad \boldsymbol{\sigma} = \rho\frac{\partial\pi}{\partial\boldsymbol{\varepsilon}^{rev}},\quad s = -\frac{\partial\pi}{\partial T} \tag{10}$$

where $\mathbf{H}$, $\boldsymbol{\sigma}$ and s are considered as thermodynamic forces associated with $\mathbf{B}$, $\boldsymbol{\varepsilon}^{rev}$ and T, respectively. By analogy, we may define a thermodynamic force conjugate to the internal variable $\boldsymbol{\xi}_1$:

$$\mathbf{R}_1 = \rho\frac{\partial\pi}{\partial\boldsymbol{\xi}_1} \tag{11}$$

For the isothermal condition the Helmholtz free energy is assumed to be

$$\rho\pi = \frac{1}{2}\left(\mathbf{B} - \mathbf{B}^{irr}\right)\cdot\boldsymbol{\mu}^{-1}\cdot\left(\mathbf{B} - \mathbf{B}^{irr}\right) + \frac{1}{2}\boldsymbol{\varepsilon}^{rev}:\mathbf{E}:\boldsymbol{\varepsilon}^{rev} + \frac{\alpha_1}{2}\boldsymbol{\xi}_1\cdot\boldsymbol{\xi}_1 \tag{12}$$

where $\boldsymbol{\mu}$ is the second-order tensor of the permeability, $\mathbf{E}$ is the fourth-order tensor of the elastic modulus, $\alpha_1 > 0$ is a material constant. The state laws for the reversible processes can be obtained by making use of Eq. (10) as shown below:

$$\mathbf{H} = \boldsymbol{\mu}^{-1}\cdot\mathbf{B}^{rev},\quad \boldsymbol{\sigma} = \mathbf{E}:\boldsymbol{\varepsilon}^{rev} \tag{13}$$

where we have used the relation of $\mathbf{B}^{rev} = \mathbf{B} - \mathbf{B}^{irr}$ from Eq. (6). By using Eq. (11), the following thermodynamic force is obtained, such that:

$$\mathbf{R}_1 = \alpha_1\boldsymbol{\xi}_1 \tag{14}$$

The Clausius–Duhem inequality is finally reduced to