Physica B 462 (2015) 104–111

Contents lists available at ScienceDirect

Physica B

journal homepage: www.elsevier.com/locate/physb

Effects of a dimple potential on the ground-state properties of a quasi-one-dimensional Bose–Einstein condensate with two- and three-body interactions

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ARTICLE INFO

Article history:
Received 16 July 2014
Received in revised form
6 January 2015
Accepted 11 January 2015
Available online 12 January 2015

Keywords:
Bose–Einstein condensation
Harmonic plus dimple potential
Ground state properties
Low-dimensional ultracold Bose gas
Nonlinear Schrödinger equation

ABSTRACT

The ground state of a quasi-one-dimensional interacting Bose gas confined by a harmonic plus Gaussian dimple potential is studied within the variational approach and also Gross–Pitaevskii mean-field approximation. The effect of the superimposed dimple trap on the order parameter, the chemical and effective potentials of the system is analyzed for repulsive and attractive two- as well as three-body interactions between the particles. The results obtained from both methods show that the characteristics of the trap such as the width and depth of the dimple affect the corresponding ground state properties of the system in a qualitatively similar way to the repulsive and attractive interatomic interactions, respectively.

1. Introduction

One of the main research interests in the sizzling field of ultracold gases is the study of low dimensionalities, where the experiments clearly lead the way [1–3]. Low-dimensional systems are also interesting from the theoretical viewpoint, since they give rise to many interesting phenomena not manifested in higher dimensions, and provide exactly solvable models, particularly in the quasi-one-dimensional case (quasi-1D) [4–9].

It has long been established that Bose–Einstein condensation does not occur in interacting homogeneous 1D systems, either at finite [10] or at zero temperature [11], due to the existence of long wavelength and quantum fluctuations, respectively. On the other hand, the presence of a trap potential may lead to Bose–Einstein condensation in the weakly-interacting limit, changing the density of states of the system at low energies [12–14].

For the 1D case, while the occurrence of Bose–Einstein condensate (BEC) depends on the presence of a trap potential, it is understood that the character of the condensate is substantially influenced by the form of the potential. Also, in such systems, the interactions between the particles play a crucial role despite the very dilute nature of these gases. The effect of atom–atom interactions in these systems paves the way for many interesting phenomena, e.g., the collapse of the condensate [15,16] or Tonks–Girardeau gas [17], depending on the strength and the nature (i.e., repulsive or attractive) of two-body interactions which can be controlled by frequently used Feshbach resonances technique [18,19]. However, if a condensate of high atomic density is in question, as in the case of atomic wave guides or atom chips, then it is necessary to take into account the three-body interactions between the particles, as well as the two-body ones. The controllability of the three-body interaction independently of its two-body counterpart further enriches the physical picture of the corresponding systems [20–24].

Moreover, it is also known that some specific distortions in the confining potential, such as the presence of a dimple, may increase the phase-space density [25] and therefore affect the properties of Bose–Einstein condensed phase [26–30].

Dimple potentials have been realized experimentally [31–33] and have a variety of applications, including, e.g., the growth of quasi-condensates on atom chips [34], the propagation of ultra-slow optical pulse in atomic BEC [35], or the production of large BECs [27]. In all these studies, the use of dimple potentials has been shown to be a good strategy to reach quantum degeneracy with many atoms in a short time and with a simple experimental setup. In this respect, it is expected that a superimposed dimple trap makes the three-body interactions between the particles important due to its effect enhancing the density of the condensate as it is observed from the experiments [31–33]. Especially, the most recent experiment of Stellmer et al. [36] shows clearly this effect where a dimple potential helps the formation of a condensate by leading to a great increase in the local atom density without increasing the temperature of the system. Following this recent experimental advent on dimple traps, Dutta and Mueller

http://dx.doi.org/10.1016/j.physb.2015.01.006
0921-4526/

have investigated theoretically the effect of the dimple trap parameters on the dynamics of a BEC [37].

Motivated by the above-mentioned recent advances on quasi-1D systems and on dimple potentials, as well as the importance of three-body interactions in high density BECs, in this paper we examine the ground-state properties of a BEC with two- and three-body interactions confined by a 1D harmonic plus Gaussian dimple potential. For 1D quantum gases, although quantum fluctuations are greatly enhanced and the effects of strong correlations between the atoms become very important, the zero-temperature properties of a weakly interacting Bose gas can be successfully described within the framework of mean-field approximation, as previously shown by several studies [38–41]. In a quasi-1D system, where all degrees of freedom except for the longitudinal direction are assumed to be frozen due to the presence of a tight transverse confinement, an effective quintic term arises in the standard Gross–Pitaevskii equation (GPE) with cubic nonlinearity as a deviation from the strictly 1D case [39,42,43]. This so-called cubic-quintic (CQ) 1D GPE provides a good description of the system in the presence of both two- and three-body interactions analog to its counterpart in 3D [44]. Here it is important to note that GP approximation is known to be exact in the thermodynamic limit for $N\to\infty$ [45–47] and as the number of particles deviates more from this limit, the finite-size effects become more important in the system that one should resort to other numerical approaches (e.g., configuration interaction method [48,49], quantum Monte-Carlo [50]) rather than the GP approach.

In the present work, we make use of the CQ GPE to study the effect of the superimposed dimple potential on the ground state properties, e.g., the order parameter, the chemical and effective potentials, of the system considering two- and three-body interactions, either repulsive or attractive, between the particles. To that end, we have performed both variational and numerical calculations. We have found that the effects of the width and depth of the dimple on the corresponding properties of the system are in analogy to those of repulsive and attractive interactions, respectively.

The paper is organized as follows. In Section 2 we present our model and method. The results we have obtained are discussed in Section 3 and a brief summary is presented in Section 4.

2. Model and method

The harmonic plus dimple potential that we consider is modeled via a combination of a 1D harmonic potential with a Gaussian dimple potential:

$$V_z(z) = \tfrac{1}{2}m\omega_z^2 z^2 - V_0 e^{-(z-z_D)^2/2\omega^2}, \tag{1}$$

where ω_z is the trap frequency of the harmonic confinement in the longitudinal direction, V_0 is the depth (strength) of the dimple trap located at $z = z_D$, and ω determines the width of the dimple. Here we assume a strong confinement along the transverse direction whose trap frequency $\omega_\perp$ satisfies $\omega_\perp \gg \omega_z$, and thus makes the motion of atoms quasi-1D along the longitudinal direction. One may then assume that the order parameter has the form

$$\Psi(\mathbf{r}, t) = \exp(-i\mu_0 t/\hbar)\Phi(\mathbf{r}_\perp)\phi(z), \tag{2}$$

where $\Phi(\mathbf{r}_\perp)$ is the ground state of the transverse harmonic potential $V_t(\mathbf{r}_\perp)$. Therefore, the problem effectively reduces from three to one dimension, and the order parameter in the z-direction $\phi(z)$ satisfies a generalized 1D GPE with the CQ nonlinearity

$$-\frac{\hbar^2}{2m}\frac{d^2\phi(z)}{dz^2} + V_z(z)\phi(z) + g_{1D}|\phi(z)|^2\phi(z) - g|\phi(z)|^4\phi(z) = \mu_0\phi(z), \tag{3}$$

where m is the atomic mass, g_{1D} is the two-body interaction strength describing the short range atom–atom interactions through s-wave scattering length a ($a > 0$ or $a < 0$ for repulsive or attractive interatomic interaction, respectively), the coefficient of the quintic term ($g = 6\ln(4/3)g_{1D}^2/\hbar\omega_\perp$) governs the strength of the three-body interaction in our model, and μ_0 is the chemical potential of the system. Here, the quintic term originates from the coupling between the longitudinal and transverse dynamics of the condensate [39,42], and can be neglected in the limit, $g \to 0$, where Eq. (3) reduces to a standard GPE with only two-body interactions. The normalization condition for ϕ is

$$\int_{-\infty}^{\infty}|\phi(z)|^2 dz = 1. \tag{4}$$

By introducing $\psi = \sqrt{l_z}\,\phi$, $x = z/l_z$, and $\omega_d = \omega/l_z$, where $l_z = \sqrt{\hbar/m\omega_z}$ is the oscillator length in the z direction, as well as by setting $z_D = 0$, Eq. (3) can be re-written in dimensionless form as

$$-\frac{d^2\psi}{dx^2} + x^2\psi - V_d e^{-\left(x/\sqrt{2}\omega_d\right)^2}\psi + \lambda|\psi|^2\psi + \varepsilon\lambda^2|\psi|^4\psi = \mu\psi, \tag{5}$$

with the dimensionless quantities $\lambda = 2g_{1D}/\hbar\omega_z l_z$, $V_d = 2V_0/\hbar\omega_z$, $\varepsilon = -(3\ln(4/3)\omega_z/\omega_\perp)$ and $\mu = 2\mu_0/\hbar\omega_z$. Here, the three-body interactions are repulsive for $\varepsilon > 0$ and are attractive for $\varepsilon < 0$.

In the presence of two- and three-body interactions, the particles in the system are subjected to the following effective potential:

$$V_{eff}(x) = x^2 - V_d e^{-\left(x/\sqrt{2}\omega_d\right)^2} + \lambda|\psi|^2 + \varepsilon\lambda^2|\psi|^4. \tag{6}$$

To solve Eq. (5), we will use two different approaches given as follows.

2.1. Variational approach

If Eq. (5) is multiplied by ψ and integrated over the whole space, one can obtain the following expression for the chemical potential:

$$\mu = \int_{-\infty}^{\infty}\left[\left(\frac{d\psi}{dx}\right)^2 + \left(x^2 - V_d e^{-\left(x/\sqrt{2}\omega_d\right)^2}\right)|\psi|^2 + \lambda|\psi|^4 + \varepsilon\lambda^2|\psi|^6\right]dx. \tag{7}$$

Similarly, the total energy of the system is expressed by

$$E = \int_{-\infty}^{\infty}\left[\left(\frac{d\psi}{dx}\right)^2 + \left(x^2 - V_d e^{-\left(x/\sqrt{2}\omega_d\right)^2}\right)|\psi|^2 + \frac{\lambda}{2}|\psi|^4 + \frac{\varepsilon\lambda^2}{3}|\psi|^6\right]dx. \tag{8}$$

The trial Gaussian wave function that we use in our variational analysis is

$$\psi(x) = \frac{1}{\pi^{1/4}\sqrt{\alpha}}e^{-\left(x^2/2\alpha^2\right)}, \tag{9}$$

where α is the dimensionless variational parameter. The root mean square radius, x_{rms}, is a function of α given below

$$x_{rms} = \sqrt{\langle x^2\rangle} = \sqrt{\frac{\alpha^2}{2}}, \tag{10}$$

and the variational central density reads

$$\rho_{c,var}(\alpha) = |\psi(0)|^2 = \frac{1}{\sqrt{\pi}\alpha}. \tag{11}$$

In order to obtain chemical potential in terms of α, we insert Eq.