Nuclear Instruments and Methods in Physics Research A 812 (2016) 33–36

Contents lists available at ScienceDirect

Nuclear Instruments and Methods in Physics Research A

journal homepage: www.elsevier.com/locate/nima

FELWI realization difficulties

Koryun B. Oganesyan

A.I. Alikhanyan National Scientific Laboratory, Yerevan Physics Institute 2 Alikhanian Br., 75036 Yerevan, Armenia,

ARTICLE INFO

Article history:
Received 30 October 2015
Accepted 19 December 2015
Available online 2 January 2016

Keywords:
Free electron laser
Lasing without inversion
Undulator
Laser threshold
Intensity
Power

ABSTRACT

For a free-electron laser without inversion (FELWI), estimates of the threshold laser power are found. The interaction induced deviation angle of electrons in the first undulator is found. It is shown that for real beams this angle is less than natural divergence angle of the beam. The large-amplification regime should be used to bring an FELWI above the threshold laser power.

1. Introduction

In a free-electron laser (FEL), coherent stimulated radiation is produced by the accelerated motion of electrons in the ponderomotive potential formed by the combined field of the wiggler and the electromagnetic wave. The so-called free-electron laser without inversion (FELWI) proposed recently [1–6] relies on the non-collinear arrangement of the electron and laser beams. Such systems are known related to usual FELs and strophotrons [7–11]. An FELWI aims to improve performance of FELs and optical klystrons and to extend operation domain to shorter wavelengths. This is to be achieved by a two-wiggler design employing an advanced laser-induced electron phasing in the first wiggler. It reveals itself in the proportionality of the laser-induced changes of the electron energy and the laser-induced angular deviation of electron velocity, which is specific to the non-collinear interaction geometry of an electron beam and laser beam. Engineering of the angular-dependent electron path length in the drift region between the two wigglers makes it possible to tune most of the electrons exiting the first wiggler closer to the phase, which is favorable for efficient amplification in the second wiggler. As a result, an FELWI gain G becomes positive for almost every detuning $\Omega \equiv \omega(v_0 - v_{res})/c$, which characterizes deviation of the electron velocity or the laser frequency from the resonance condition, $\Omega = \int G(\Omega)\, d\Omega > 0$ [1–6].

2. Single particle approximation

According to the main idea of Ref. [4], a possibility of FELWI realization is strongly related to a deviation of electrons from their original direction of motion owing to interaction with the fields of undulator and co-propagating light wave. The deviation angle appears to be proportional to energy gained or lost by an electron during its passage through the undulator. Owing to this, a subsequent regrouping of electrons over angles provides regrouping over energies. In principle, a proper installation of magnetic lenses and turning magnets after the first undulator in FELWI can be used in this case for making faster electrons running over a longer trajectory than the slower ones [5]. This is the *negative-dispersion* condition which is necessary for getting *amplification without inversion* [1].

It is clear that the described mechanism can work only if the interaction-induced deviation of electrons (with a characteristic angle $\Delta\alpha$) is larger than the natural angular width $\alpha_{\rm beam}$ of the electron beam (see Fig. 1),

$$\Delta\alpha > \alpha_{\rm beam}. \tag{1}$$

As the energy gained/lost by electrons in the undulator and the deviation angle are proportional to the field strength amplitude of the light wave to be amplified, condition (1) determines the threshold light intensity, only above which amplification without inversion can become possible. This threshold intensity is estimated below.

In the non-collinear FEL the electron slow-motion phase is defined as

$$\varphi = qz + \vec{k} \cdot \vec{r} - \omega t, \tag{2}$$

E-mail address: bsk@yerphi.am

http://dx.doi.org/10.1016/j.nima.2015.12.038
0168-9002/

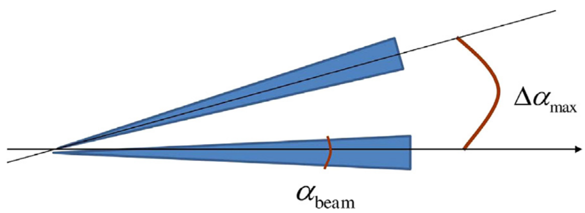

Fig. 1. The scheme of electron beam after first wiggler.

where $q = 2\pi/\lambda_0$ and λ_0 is the undulator period, $\vec{k}$ and ω are the wave vector and frequency of the wave to be amplified, respectively, $|\vec{k}| = \omega/c$, $\vec{r} = \vec{r}(t)$ is the electron position vector and $z = z(t)$ is its projection on the undulator axis. Let the initial electron velocity $\vec{v}_0$ be directed along the undulator axis $0z$. Let the undulator magnetic field $\vec{H}$ be directed along the x-axis. Let the light wave vector $\vec{k}$ be lying in the (xz) plane under an angle θ to the z-axis. Let the electric field strength $\vec{\varepsilon}$ of the wave to be amplified is directed along the y-axis, as well as its vector potential $\vec{A}_{\text{wave}}$ and the undulator vector potential $\vec{A}_{\text{und}}$, where

$$A_{\text{wave}} = \frac{c\varepsilon_0}{\omega}\cos\left(\vec{k}\cdot\vec{r} - \omega t\right), \quad A_{\text{und}} = \frac{H_0}{q}\cos(qz), \tag{3}$$

and ε_0 and H_0 are the amplitudes of the electric component of the light field and of the undulator magnetic field, respectively. The described geometry corresponds to that considered in Ref. [4]. The slow motion phase (2) obeys the usual pendulum equation

$$\ddot{\varphi} = -a^2 \sin\varphi, \tag{4}$$

where

$$a = \frac{ce\sqrt{\varepsilon_0 H_0}}{E_0}; \tag{5}$$

$E_0 \equiv \gamma mc^2$ is the initial electron energy and γ is the relativistic factor. If L is the undulator length, the ratio L/c is the time it takes for an electron to pass through the undulator. The product of this time by the parameter a of Eq. (5) is known [10] as the saturation parameter μ,

$$\mu = \frac{aL}{c} = \frac{eL\sqrt{\varepsilon_0 H_0}}{E_0}. \tag{6}$$

Amplification in FEL (with $H_0 = const$) is efficient one as long as $\mu \le 1$. At $\mu > 1$ the FEL gain G falls. The condition $\mu \sim 1$ determines the saturation field $\varepsilon_{0\,\text{sat}}$ and intensity I_{sat}. For example, at $L = 3$ m, $H_0 = 10^4$ Oe, $\gamma = 10^2$ we have $\varepsilon_{0\,\text{sat}} \sim 1.2 \times 10^4$ V/cm and $I_{\text{sat}} \sim 2\times 10^5$ W/cm^2. In our further estimates of the FELWI threshold field and intensity we will have to keep in mind that it is hardly reasonable to consider fields stronger than the saturation field $\varepsilon_{0\,\text{sat}}$.

In accordance with the results of Refs. [4, Eq. (14)] and [5, Eq. (13)], a transverse velocity v_x and energy ΔE acquired by an electron after a passage through the undulator are directly proportional to each other:

$$v_x = c\,\theta\frac{\Delta E}{E_0}, \tag{7}$$

which gives in the first order the following estimate of the electron deviation angle $\Delta\alpha$:

$$\Delta\alpha \approx \frac{v_x^{(1)}}{v_0} \approx \frac{v_x^{(1)}}{c} = \theta\frac{\Delta E^{(1)}}{E_0} \sim \theta\,\mu^2\frac{\lambda_0}{4\pi L} \sim \mu^2\frac{d\lambda_0}{4\pi L^2}, \tag{8}$$

where d is the electron beam diameter and we took $\theta \sim d/L$. Here we have used the first-order change of the electron energy $\Delta E^{(1)}$ Eq. (13) of [13]:

$$\Delta E^{(1)} = \mu^2 E_0\frac{\lambda_0}{4\pi L}. \tag{9}$$

Let us take for estimates maximal value of the saturation parameter μ compatible with the weak-field approximation, $\mu \sim 1$. Let us take also $\lambda_0 = 3$ cm, $d = 0.3$ cm, and $L = 3\times 10^2$ cm. Then, we get from Eq. (8) the following estimate of the electron deviation angle:

$$\Delta\alpha \sim 10^{-6}. \tag{10}$$

3. Collective description

The interaction of electron beam with laser field can be described by laws of conservation for momentum $\mathbf{p}_e + \mathbf{p}_L = \mathbf{p}'_e + \mathbf{p}'_L$ and energy $\mathcal{E}_e + \mathcal{E}_L = \mathcal{E}'_e + \mathcal{E}'_L$. Here $\mathbf{p}_e$ and $\mathbf{p}'_e$ are initial and final momentums of electrons, respectively; $\mathbf{p}_L$ and $\mathbf{p}'_L$ are initial and final momentums of laser field, respectively; $\mathcal{E}'_L$ and $\mathcal{E}_L$ are initial and final energies of light beam, respectively, and $\mathcal{E}'_e$ and $\mathcal{E}_e$ are initial and final energies of electrons, respectively. The density of electromagnetic wave momentum is $\mathbf{P}_L = 1/(4\pi c)[\mathbf{EB}] = \mathbf{k}\omega/(4\pi c)\mathbf{A}_L^2$, where $\mathbf{A}_L$ is an amplitude of a vector-potential of laser field. We can write for $\mathbf{A}'_L = \mathbf{A}_L \exp(k'' L)$, where k'' is a spatial growth rate of laser field in a medium of an electron beam; L is a length of interaction. From law of conservation we can expect that $|\Delta\mathbf{p}| = |\mathbf{p}'_e - \mathbf{p}_e| = |\mathbf{p}'_L - \mathbf{p}_L| = \mathbf{A}_L^2[\exp(2k'' L) - 1]$. We can see that the change of electron momentum $|\Delta\mathbf{p}|$ depends on the spatial growth rate k'': with the growth rate k'' rising, the change of electron momentum rises too. This means that for noncollinear interaction the deviation of electron from its original direction depends on both the spatial growth rate k'' and the amplitude A_L of laser field. The growth rate k'' is a function on electron beam current; and the amplitude depends on laser power.

3.1. Space amplification

We consider the induced radiation by a mono-energetic beam of electrons propagating in a wiggler. We assume that the static magnetic field of a plane undulator $\mathbf{A}_w$ is independent of the transverse coordinates x and y. Also we approximate the static magnetic field by a harmonic function $\mathbf{A}_w = A_w\mathbf{e}_y = (A_0 e^{-i\mathbf{k}_w\mathbf{r}} + \text{c.c.})\mathbf{e}_y$, where $\mathbf{k}_w = (0, 0, k_w)$ is the wiggler wave vector; “c.c.” denotes the complex conjugation, and $\mathbf{e}_y$ is the unit vector along the y-axis. The wiggler field causes an electron to oscillate along the y-axis. For this reason, the electron interacts most efficiently with a light wave if the latter is linearly polarized. We assume that the vector potential of the laser wave has a linear polarization $\mathbf{A}_L = A_L(t, x, z)\mathbf{e}_y = \mathbf{a}_+ e^{i(\mathbf{k}-\mathbf{k}_w)\mathbf{r} - i\omega t}$.

The early theoretical constructions assume infinite electron and laser beams. In reality both electron and laser beams are restricted in the transverse directions. This means that the non-collinear arrangement of electron and laser beams leads to the finite area of their interaction (Fig. 2). The length of laser amplification in the medium of electron beam is $L_L = 2r_b/\sin(\alpha + \theta)$, where $2r_b$ is a width of the electron beam in the xz-plane (Fig. 2). The length, at which the electrons move acting by force of laser field, is equal to $L_e = 2r_L/\sin(\alpha + \theta)$, where $2r_L$ is a width of the laser beam in the xz-plane.

3.2. The FELWI threshold

The solution of the linearized equations for slow motion of the electron in the xz-plane is [12]:

$$\delta\mathbf{v}_{\parallel} = K^2\frac{c^2}{\gamma_0^3}\sum_{j=1}^{4}\frac{\beta_1\mathbf{k}_j - \frac{\omega}{c^2}\beta_2\mathbf{u}}{D_{b(j)}}a_j e^{i\xi_0 - i\Delta_{\omega(j)}t} + \text{c.c.} \tag{11}$$

Here K is the undulator strength parameter, defined as normalized dimensionless vector-potential of the undulator magnetic field $K = e/(mc^2)|A_0|$. The total relativistic factor of electrons γ_0 is defined as $\gamma_0 = \sqrt{1 + 2K^2}(1 - u^2/c^2)^{-1/2}$, where the initial velocity