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Impact of colored noise in pulse amplitude measurements: A time-domain approach using differintegrals

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ABSTRACT

In particle detectors, pulse shaping is the process of changing the waveform of the pulses in order to maximize the signal to noise ratio. This shaping usually only takes into account white, pink (flicker) and red (Brownian) noise. In this paper, a generalization of noise indexes as a function to an arbitrary f^{β} noise type, where β is a real number, is presented. This generalization has been created using the differintegral operator, defined in Fractional Calculus. These formulas are used to calculate the Equivalent Noise Change (ENC) in detector particle systems.

1. Introduction

In spectroscopy systems, pulse shaping plays a crucial role in noise filtering. In order to analyze different shaping modes, Goulding [1] and Radeka [2] defined the noise indexes of shapers (also called "form factors" in [3]) as parameters proportional to the contribution of a specific noise type. These parameters only depend on the pulse shape and its duration. A different noise index has to be calculated for each different "color of noise". In a signal with components at all frequencies and a power spectral density per unit of bandwidth proportional to f^{β}, the color is given by the β value. For instance, the spectral density of white noise is flat ($\beta = 0$), while pink (flicker) noise has $\beta = -1$ and red (Brownian) noise has $\beta = -2$.

In this paper, all the noise spectral densities are referred to the preamplifier output. Goulding [1] calculated the noise indexes for voltage (white) and current (red) noise at this point of the circuit. In [4] the f^{-1} (pink) noise index using the concept of 1/2-derivative developed in Fractional Calculus [5] was also introduced. A strength of noise indexes is that they are calculated in time-domain directly whereas other methods that use Fourier Transforms are less intuitive and more complex to carry out. The first conclusion taken from the noise indexes is that the contribution from red noise increases with shaping time whereas the white noise contribution decreases. The f^{-1} noise does not depend on the shaping time. Fig. 1 shows a typical example of ENC at shaper output vs. shaping time in presence of red and white noise.

Until now, noise analysis have been performed just for white, pink and red noise (e.g. [6]), which are proportional to f^{-2}, f^{-1} and f^{0} respectively. However, in particle detectors, noise distribution is often more complex. In fact, the most common noise in particle detectors has a continuous range from $f^{-0.5}$ to f^{-2} [7,8]. In this paper, a generalization of the noise indexes using differintegrals is proposed with the aim of covering a continuous desired range, instead of using only discrete values such as f^{-2}, f^{-1} or f^{0}. With this generalization, shapers can be analyzed more deeply.

In principle, this analysis can be used to obtain the generalized noise parameters of a shaper. This analysis can be used individually, or as a cost function of an automated algorithm to find the optimal shaping. Moreover, this method also allows analyzing a shaper, provided by optimization algorithms, to find the predominant noise type present in the system, and then try to mitigate it. There is extensive material published on optimal pulse shaping synthesis (e.g. [9–12]).

Finally, we would like to clarify that this paper focuses on noise impact measurement, but does not focus on selecting the most suitable pulse shape for a given spectroscopy system or particle detector; instead, in this paper we describe a method to analyze the relative noise performance of pulse-shaping systems.

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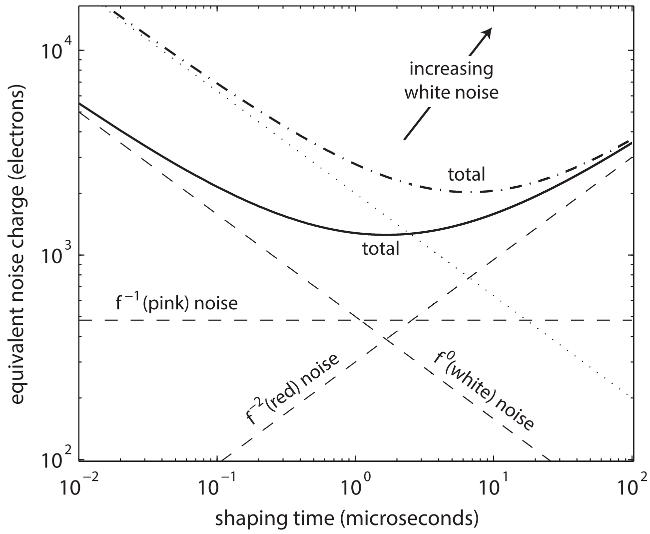

Fig. 1. Equivalent noise charge vs. shaping time. Changing the red noise ($\beta = -2$) or, as in this case, white noise ($\beta = 0$) contribution shifts the noise minimum. Increased voltage noise is shown as an example. (Figure reproduced from [3]) with permission.

2. Differintegrals

Whenever a function $W(t)$ is derived n (positive integer) times or integrated $-n$ times, we can replace n for a real number α. If $\alpha > 0$, $W^{(\alpha)}(t)$ is the α fractional derivative of $W(t)$. Otherwise, $W^{(\alpha)}(t)$ is the $-\alpha$th fractional integral. Differintegrals are a combined fractional differentiation/integration operator. Therefore, $W^{(\alpha)}(t)$ is the Differintegral operator [5] applied to $W(t)$. Actually, α can be also a imaginary number [13] leading to complex-order derivatives. However, for our purposes, it is sufficient that α be a real number.

In the literature, there are several definitions of fractional derivative and integral [14]. Thus, to define the differintegral operator, it must be defined first fractional derivatives and integrals separately.

On one hand, the classical form of fractional integral is the Riemann–Liouville definition:

$$J^{\alpha}f(t) := \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau)\, d\tau \tag{1}$$

where α is a real positive number, Γ is the Gamma Function and J is the Riemann–Liouville integral operator.

On the other hand, the definition of Riemann–Liouville fractional derivative is based in the previous formula and is given by

$$D^{\alpha}f(t) := \frac{1}{\Gamma(n-\alpha)}\frac{d^n}{dt^n}\left(\int_0^t (t-\tau)^{n-\alpha-1} f(\tau)\, d\tau\right) \tag{2}$$

where n is an integer number. This equation is the cornerstone of fractional calculus.

Although both operators are linear, J commutes (i.e. $J^{\alpha}J^{\beta}f(t) = J^{\beta}J^{\alpha}f(t)$). However, D does not commute for non-integer numbers, that is $J^{\alpha}D^{\alpha}f(t) \neq D^{\alpha}J^{\alpha}f(t)$. In addition $D^{\alpha}k$ for any constant k is not always equal to 0. To solve these drawbacks, alternative definitions for fractional derivatives were proposed. One of the most popular is the Caputo derivative, also based on Eq. (1):

$$D_c^{\alpha}f(t) := J^{\lceil\alpha\rceil-\alpha}D^{\lceil\alpha\rceil}f(t) \tag{3}$$

where $\lceil\alpha\rceil$ is the ceiling function, which provides the smallest integer greater than or equal to α. Then, in this case, the value of $D^{\lceil\alpha\rceil}f(t)$ is a derivative of integer value. This new operator is linear and commutes, that is $J^{\alpha}D_c^{\alpha}f(t) = D_c^{\alpha}J^{\alpha}f(t)$, and $D_c^{\alpha}k = 0$ for any constant k. Both operators, J and D_c form the differintegral operator. However, both J and D_c are complex to calculate by means of numerical methods.

To approximate the value of the differintegral, instead of J and D_c operators, in this paper and henceforth we are going to use the Grünwald–Letnikov definition given by

$$f^{(\alpha)}(t) = \lim_{h\to 0}\frac{1}{h^{\alpha}}\sum_{j=0}^{k}(-1)^j\binom{\alpha}{j}f(kh-jh) \tag{4}$$

This formula is easily implemented using numerical methods [16] compared to (1) and (3) and it has been used in another works related to filters and numerical calculus (e.g. [17]).

3. Generalization of the ENC formula

As a starting point, we are going to use the ENC formula presented in [8,3] because it is necessary to know the noise indexes to be calculated. The ENC formula is

$$Q_n^2 = i_n^2 F_i \tau_s + v_n^2 F_v \frac{C^2}{\tau_s} + F_{vf} A_f C^2 \tag{5}$$

where Q_n is the ENC in Coulombs, τ_s is the total shaping time and C is the equivalent detector capacitance. F_v, F_i, and F_{vf} are the noise indexes for f^0-noise, f^{-2}-noise and f^{-1}-noise, respectively; in this nomenclature, they are dimensionless. i_n is the current noise spectral density measured in $\mathrm{A}/\sqrt{\mathrm{Hz}}$, v_n is the voltage noise spectral density measured in $\mathrm{V}/\sqrt{\mathrm{Hz}}$, A_f is the f^{-1}-noise spectral density coefficient measured in V^2. The f^{-1}-noise spectral density v_{nf} is equal to

$$v_{nf} = \sqrt{\frac{A_f}{f}}[\mathrm{V}/\sqrt{\mathrm{Hz}}] \tag{6}$$

Other nomenclatures different than the one proposed in [3] such as [15,8] are equivalent. Equation (5) is applicable to both analog and digital shapers.

The value of F_i and F_v are

$$F_i = \frac{1}{2\tau_s}\int_{-\infty}^{\infty} W^2(t)\, dt \tag{7}$$

$$F_v = \frac{\tau_s}{2}\int_{-\infty}^{\infty} (W'(t))^2\, dt \tag{8}$$

where for time-invariant pulse shaping $W(t)$ is the system's impulse response for a short input pulse with the peak output signal normalized to unity. For time-variant systems (e.g. gated integrators), $W(t)$ can be also easily calculated according to the method described in [1]. An alternative notation of these last two formulas can be found in the same reference.

The expression for F_{vf} can be deduced from [15,4] and is equal to

$$F_{vf} = \frac{1}{2}\int_{-\infty}^{\infty}\left(W^{(1/2)}(t)\right)^2 dt \tag{9}$$

where $W^{(1/2)}(t)$ is the 1/2-derivative of $W(t)$. It must be taken into account that the calculus of the 1/2-derivative in time domain is equivalent to multiply by $\sqrt{s}$ in Laplace domain. There are several methods (analytical and numerical) to calculate the fractional derivatives [5]. One of the simplest for 1/2-derivative calculation was proposed in [4]:

$$W^{(1/2)}(t) = \frac{1}{\sqrt{\pi t}} * W'(t),\ \forall t > 0 \tag{10}$$