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An exact solution of spherical mean-field plus a special separable pairing model

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Abstract

An exact solution of nuclear spherical mean-field plus a special orbit-dependent separable pairing model is studied, of which the separable pairing interaction parameters are obtained by a linear fitting in terms of the single-particle energies considered. The advantage of the model is that, similar to the standard pairing case, it can be solved easily by using the extended Heine–Stieltjes polynomial approach. With the analysis of the model in the ds- and pf-shell subspace, it is shown that this special separable pairing model indeed provides similar pair structures of the model with the original separable pairing interaction, and is obviously better than the standard pairing model in many aspects.

Keywords: Separable pairing interaction; Exact solutions; Shell model

1. Introduction

Pairing interaction has been considered of importance in many branches of physics [1–4]. In nuclear physics, the pairing interaction is key to elucidate ground state and low-energy spectroscopic properties of nuclei, such as binding energies, odd-even effects, single-particle occupancies, excitation spectra, and moments of inertia, etc. [5–7]. Though the Bardeen–Cooper–Schrieffer (BCS) [1] and the more refined Hartree–Fock–Bogolyubov (HFB) approximations

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provide simple and clear pictures in nuclei [6,8,9], tremendous efforts have been made in finding accurate solutions to the problem [10–15] to overcome serious drawbacks in the BCS and the HFB resulting from particle number-nonconservation effects [13,15–17]. It is known that either spherical or deformed mean-field plus the standard orbit independent pairing interaction can be solved exactly by using the Gaudin–Richardson method [18–20]. It has been shown that the set of Gaudin–Richardson equations can be solved easily by using the extended Heine–Stieltjes polynomial approach [21–24]. The deformed and spherical mean-field plus the extended pairing models have also been proposed, which can be solved more easily than the standard pairing model, especially when both the number of valence nucleon pairs and the number of single-particle orbits are large [25,26].

The separable pairing problem was studied in [27], in which the single-particle energies are all degenerate. A case with two non-degenerate levels was analyzed in [28]. General non-degenerate cases were also considered previously [29,30]. However, the expressions are complicated with the specially chosen constraints on the single-particle energies and the separable pairing interaction parameters, which are difficult to be solved numerically. In this work, we show that there are special cases of [29,30], of which the solutions can be obtained easily by using the algebraic Bethe ansatz. It should be pointed out that solutions of the special separable pairing cases considered in this work were also derived previously by using the generalized Gaudin algebraic formalism [31,32]. These special cases belong to a special family of the hyperbolic Richardson–Gaudin models studied in [31,32].

2. The model and its exact solution

The Hamiltonian of a mean-field plus the separable pairing model may be written as

$$\hat{H} = \sum_{t=1}^{p} \epsilon_{j_t} \hat{N}_{j_t} + \hat{H}_{\mathrm{P}} = \sum_{t=1}^{p} \epsilon_{j_t} \hat{N}_{j_t} - G \sum_{1 \le t, t' \le p} c_{j_t} c_{j_{t'}} S^{+}_{j_t} S^{-}_{j_{t'}}, \tag{1}$$

where p is the total number of orbits considered, $\{\epsilon_{j_t}\}$ $(t = 1, 2, \cdots, p)$ is a set of single-particle energies generated from any mean-field theory, such as those of the shell model, $\hat{N}_j = \sum_m a^{\dagger}_{jm} a_{jm}$ and $S^{+}_{j} = \sum_{m>0} (-1)^{j-m} a^{\dagger}_{jm} a^{\dagger}_{j-m}$, in which $a^{\dagger}_{jm}$ (a_{jm}) is the creation (annihilation) operator for a nucleon with angular momentum quantum number j and that of its projection m, G and $\{c_{j_t}\}$ $(t = 1, 2, \cdots, p)$ are the separable pairing interaction parameters, which are all assumed to be real in the following.

The set of local operators $\{S^{-}_{j_t},\ S^{+}_{j_t},\ \hat{N}_{j_t}\}$ $(t = 1, 2, \cdots, p)$, where $S^{-}_{j_t} = (S^{+}_{j_t})^{\dagger}$, generate p copies of SU(2) algebra satisfying the commutation relations

$$[\hat{N}_{j_t}/2,\ S^{-}_{j_{t'}}] = -\delta_{tt'} S^{-}_{j_t},\ [\hat{N}_{j_t}/2,\ S^{+}_{j_{t'}}] = \delta_{tt'} S^{+}_{j_t},\ [S^{+}_{j_t},\ S^{-}_{j_{t'}}] = 2\delta_{tt'} S^{0}_{j_t}, \tag{2}$$

where $S^{0}_{j_t} = (\hat{N}_{j_t} - \Omega_{j_t})/2$ with $\Omega_{j_t} = j_t + 1/2$.

Let

$$S^{+}(x) = \sum_{t=1}^{p} \frac{1}{2\epsilon_{j_t} - x} c_{j_t} S^{+}_{j_t}, \tag{3}$$

where x is the spectral parameter to be determined, which was used in [29,30]. According to the commutation relations given in (2), we have