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Entanglement entropy of de Sitter space α-vacua

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Abstract

We generalize the analysis of [1] to de Sitter space α-vacua and compute the entanglement entropy of a free scalar for the half-sphere at late time.

1. Introduction

De Sitter space is a very interesting space–time. It is a solution of Einstein equation when cosmological constant dominates, and it is related to the inflationary stage of our universe and also current stage of accelerating universe. One peculiar property of de Sitter space is that de Sitter invariant vacuum is not unique; it has a one-parameter family of invariant vacuum states $|\alpha\rangle$, called α-vacua.

The α-vacua give very peculiar behavior for the two point functions in de Sitter space; The two point functions on α-vacua between point x and y contain not only the usual short distance singularity $\delta(|x-y|)$, where $|x-y|$ is de Sitter invariant distances between x and y, but also contain very strange singularity such as $\delta(|x-\bar{y}|)$ and $\delta(|\bar{x}-y|)$, where $\bar{x}$, $\bar{y}$ represent the antipodal points of x, y. Since antipodal points in de Sitter space are not physically accessible due to the separation by a horizon, one cannot have an immediate reason to discard two point

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functions containing such an antipodal singularity (see [2] for a nice review, and also [3,4]). It is therefore unclear which vacuum should be realized in our universe. As a result, a lot of studies have been done on phenomenological aspects of the α-vacua (e.g. primordial perturbations generated during inflation).

Since which vacuum one should choose is always a very important question, one is motivated to calculate physical quantities not only in a particular vacuum but also in others, and see if there is a deep reason to choose or discard a particular vacuum. In this letter we compute the entanglement entropy in de Sitter α-vacua. By generalizing the recent calculation by Maldacena and Pimentel [1] in the Euclidean (or Bunch–Davies) vacuum for free scalar fields, we discuss how entanglement entropy depends on α.

2. α-vacua of de Sitter space

We first introduce the α-vacua of de Sitter space in this section. Let us consider a free real scalar field Φ of the effective square-mass m^2 on de Sitter space

$$I = -\frac{1}{2}\int d^4x\sqrt{-g}\left(\partial_\mu\Phi\partial^\mu\Phi + m^2\Phi^2\right). \tag{1}$$

If we expand the scalar field $\Phi(x)$ in terms of the Euclidean vacuum mode function $\phi_n(x)$ as

$$\Phi(x) = \sum_n \left(\phi_n(x)a_n + \phi_n^*(x)a_n^\dagger\right), \tag{2}$$

the Euclidean vacuum $|0\rangle$ is defined by a state satisfying

$$a_n|0\rangle = 0. \tag{3}$$

Here $*$ represents the complex conjugate and $\dagger$ is the Hermitian conjugate. The operators $a_n^\dagger$ and a_n are the creation and annihilation operators on the Euclidean vacuum, respectively.

In analogy with (3), we can introduce a class of states annihilated by linear combinations of a and $a^\dagger$

$$\tilde{a}_n = (\cosh\alpha)a_n - e^{-i\beta}(\sinh\alpha)a_n^\dagger, \tag{4}$$

where the real parameters α and β do not depend on the label n of frequency modes. In terms of the operator (4), we introduce a two-parameter family of states defined by

$$\tilde{a}_n|\alpha,\beta\rangle = 0. \tag{5}$$

This class of states is called the α-vacua and it is known that they reproduce de Sitter invariant Green functions.

3. Entanglement entropy on α-vacua

In this section we discuss entanglement on the α-vacua of de Sitter spacetime. Using the same setup and methodology as in the Euclidean vacuum case [1], we investigate entanglement at the future infinity. After clarifying our setup, we evaluate the density matrix and the entanglement entropy on the α-vacua of free real scalar fields.