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2D quantum gravity at three loops: A counterterm investigation

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Abstract

We analyze the divergences of the three-loop partition function at fixed area in 2D quantum gravity. Considering the Liouville action in the Kähler formalism, we extract the coefficient of the leading divergence $\sim A\Lambda^2(\ln A\Lambda^2)^2$. This coefficient is non-vanishing. We discuss the counterterms one can and must add and compute their precise contribution to the partition function. This allows us to conclude that every local and non-local divergence in the partition function can be balanced by local counterterms, with the only exception of the maximally non-local divergence $(\ln A\Lambda^2)^3$. Yet, this latter is computed and does cancel between the different three-loop diagrams. Thus, requiring locality of the counterterms is enough to renormalize the partition function. Finally, the structure of the new counterterms strongly suggests that they can be understood as a renormalization of the measure action.

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1. Introduction

The coupling of conformal matter and two-dimensional quantum gravity is a subject which has been deeply studied, with a broad variety of approaches, from the discrete – triangulations [1] and matrix models [2–5] – to the continuum approaches [6–8]. In the continuum approach, most of the computations have been done within the conformal gauge. When the conformally

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coupled matter is integrated out, one ends up with the Liouville action as an effective gravity action. An interesting object to characterize is the partition function at fixed area $Z[A]$, where A is the area of a Riemann surface of genus h. In the conformal gauge, $g = e^{2\sigma} g_0$, where σ is the conformal factor and g_0 a background metric, $Z[A]$ may be formally written as

$$Z[A] = \int \mathcal{D}\sigma \ \exp\left(-\frac{\kappa^2}{8\pi} S_{\mathrm{L}} - S_{\mathrm{cosm}}\right) \delta\left(A - \int \mathrm{d}^2x \sqrt{g_0}\, e^{2\sigma}\right)$$

where $\kappa^2 = \frac{26-c}{3}$. One of the main difficulties when computing the partition function lies in the complicated non-flat measure $\mathcal{D}\sigma$ for the conformal factor.

Knizhnik, Polyakov and Zamolodchikov (KPZ) were the first to characterize this partition function at fixed area [6] for a two-dimensional quantum gravity. They derived the scaling law

$$Z[A] \sim e^{-\mu_c^2 A} A^{\gamma_{\mathrm{str}}-3}$$

and a formula for the string susceptibility γ_{str}, in the light-cone gauge for genus zero. Working in the conformal gauge, [7] and [8] managed to generalize the KPZ formula for surfaces of arbitrary genus, making several simplifying assumptions and using consistency conditions:

$$\gamma_{\mathrm{str}} = 2 + 2\,(h-1)\,\frac{\sqrt{25-c}}{\sqrt{25-c} - \sqrt{1-c}}\,.$$

While alternative derivations such as [9] and [10] for $c \leq 1$ and $h = 0$ have more recently been obtained, no obvious way to circumvent the so-called "$c = 1$ barrier", stating that this formula turns complex for $1 < c < 25$, has yet been found.

The recent development of efficient multi-loop regularization methods on curved space-times [11] opened the way for a precise and well-defined perturbative computation of this fixed-area partition function in the Kähler formalism where the conformal factor is traded for the (Laplacian of the) Kähler potential as the basic quantum field. In [12] the string susceptibility was computed in this framework up to one loop for surfaces of arbitrary genus using a somewhat more general quantum gravity action including the Liouville and Mabuchi actions; the latter corresponds to possible couplings to non-conformal matter. Of course, for conformal matter only, the one-loop KPZ result was reproduced. This was to be expected since the non-trivial nature of the quantum gravity integration measure only shows up at two and higher loops.

In [13] this computation was then extended to two loops with the Liouville action only. The regularized fixed-area partition function depends on the cut-off Λ and the area A through divergent terms of the form $A\Lambda^2$, $\ln A\Lambda^2$, $\left(\ln A\Lambda^2\right)^2$ and $A\Lambda^2 \ln A\Lambda^2$. While the first term only contributes to the divergent cosmological constant (which can be adjusted by a corresponding local counterterm), and the coefficient of the second term determines γ_{str}, the third and fourth terms are unwanted, non-local divergences. Quite non-trivially, all contributions to the third term added up to zero! However, this was not the case for the $A\Lambda^2 \ln A\Lambda^2$ divergences which remained. As carefully argued in [13] one can and must introduce local counterterms other than just the cosmological constant. Such local counterterms then also contribute, via one-loop diagrams, to the two-loop partition function. In particular, they can cancel the $A\Lambda^2 \ln A\Lambda^2$ divergences, but they could not cancel any $\left(\ln A\Lambda^2\right)^2$ divergences. Happily, the latter canceled among themselves without any need of counterterms. The precise coefficients of the counterterms were determined up to regulator-independent finite constants by requiring that the two-point function of the Kähler fields, or equivalently of $\langle e^{2\sigma} e^{2\sigma} \rangle$, be finite and regulator-independent. Their contributions to the partition function was exactly the one required to make it finite and regulator-independent. Yet,

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