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Initial states in integrable quantum field theory quenches from an integral equation hierarchy

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Abstract

We consider the problem of determining the initial state of integrable quantum field theory quenches in terms of the post-quench eigenstates. The corresponding overlaps are a fundamental input to most exact methods to treat integrable quantum quenches. We construct and examine an infinite integral equation hierarchy based on the form factor bootstrap, proposed earlier as a set of conditions determining the overlaps. Using quenches of the mass and interaction in Sinh-Gordon theory as a concrete example, we present theoretical arguments that the state has the squeezed coherent form expected for integrable quenches, and supporting an Ansatz for the solution of the hierarchy. Moreover we also develop an iterative method to solve numerically the lowest equation of the hierarchy. The iterative solution along with extensive numerical checks performed using the next equation of the hierarchy provides a strong numerical evidence that the proposed Ansatz gives a very good approximation for the solution.

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1. Introduction

The study of quantum dynamics in one-dimensional integrable systems has led to intriguing discoveries, like the experimental observation of lack of thermalization [1–3], the theoretical prediction [4] and experimental observation [5] of an unconventional statistical ensemble known as the *Generalized Gibbs Ensemble* (GGE), the discovery of unexpected effects on quantum transport [6,7] and of novel quasi-local conserved charges [8–10]. A typical protocol employed for the preparation of a closed quantum system in an out-of-equilibrium initial state is the instantaneous change of some parameter of its Hamiltonian, a process called *quantum quench* [11,12]. After a quantum quench, the initial state of the system, which is typically the ground state of the pre-quench Hamiltonian, evolves unitarily under a different post-quench Hamiltonian.

Obviously in order to derive the time evolution of the system we generally need to know the post-quench excitation amplitudes in the initial state, i.e. the overlaps of the initial state with any post-quench energy eigenstates. Determining the excitation content of the initial state is typically easy for quantum quenches in which both the pre- and the post-quench Hamiltonians are quadratic in terms of some suitable physical fields. In this class of problems, which correspond to non-interacting models or interacting models that can be mapped into non-interacting ones, the relation between pre-quench and post-quench excitations is typically described by a so-called *Bogoliubov transformation* and the initial state is then a squeezed coherent state, more precisely a *squeezed vacuum* state, of the post-quench Hamiltonian [13–15]. Such states consist of pairs of excitations with opposite momenta and have the characteristic exponential form

$$|S\rangle = \exp\left(\int_0^\infty dk\; K(k) A^\dagger(-k) A^\dagger(k)\right) |0\rangle \, , \tag{1.1}$$

where $A^\dagger(k)$ are operators that create particles of momentum k and $|0\rangle$ is the corresponding vacuum state. However, the task of determining the excitation content of the initial state is far more difficult in the context of genuinely interacting integrable systems (except in special cases [16]). The reason is that the pre-quench and post-quench excitations are no longer related through a simple linear transformation, but instead a nonlinear one that in general corresponds to an infinite series [17]. Even though the initial state has been derived exactly in a number of special cases [18–23], a general and systematic method for its determination remains so far elusive. On the other hand, while in those earlier studies the initial state was derived by means of finite volume calculations based on the Bethe Ansatz, in fact we are mostly interested in the thermodynamic limit where all finite size effects vanish and integrable quantum field theoretic methods come into play [17,24–27]. In particular, it has been argued [28] that in the thermodynamic limit, i.e. in the limit of large system size L and particle number N with a fixed density of particles, it is sufficient to know only the extensive part of the logarithmic overlaps between the initial state $|\Psi\rangle$ and the post-quench eigenstates $|\Phi\rangle$, i.e. the quantity

$$\mathcal{E}_\Psi[\Phi] = \lim_{N,L\to\infty} N^{-1} \log\langle\Phi|\Psi\rangle \, . \tag{1.2}$$

Indeed it was shown in [28] that a single post-quench eigenstate that is representative of the initial state, is sufficient for the description of the asymptotic values of local observables at times $t \to \infty$. Such a representative state is completely determined by the quantity $\mathcal{E}_\Psi[\Phi]$ above, moreover, the full time evolution can also be derived from the same quantity. The argument is based on the fundamental idea of statistical physics that microstates can be classified according to their